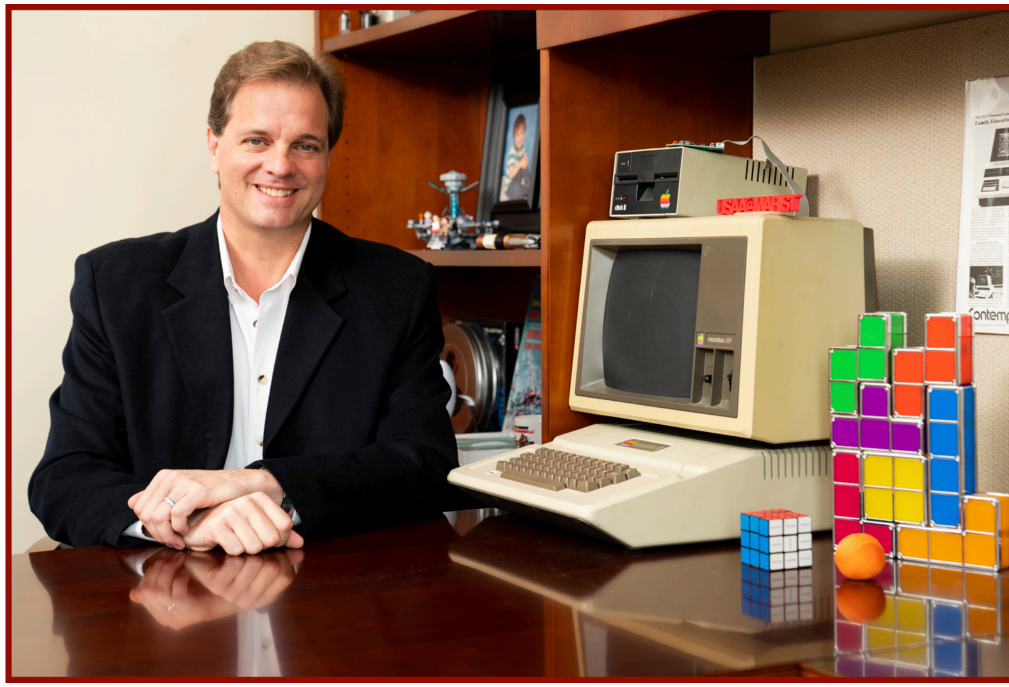
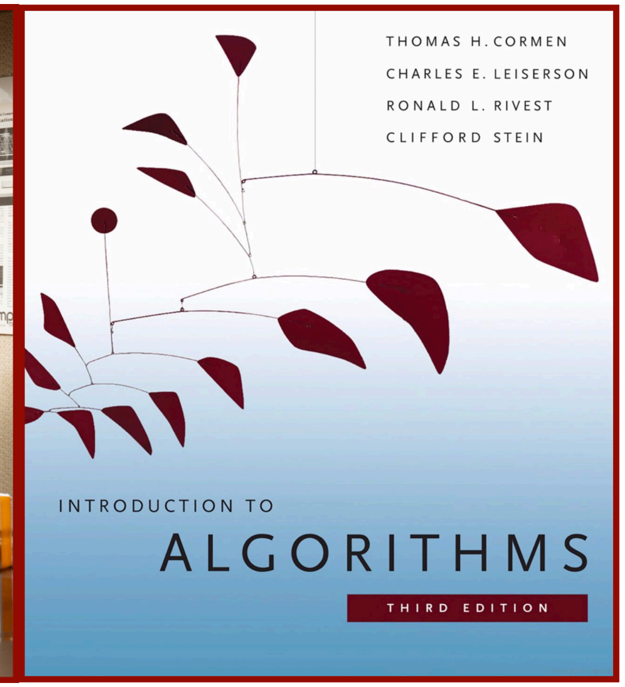
Sorting - part one





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Total Order / Linear Order

From Appendix B in our CLRS text:

A relation R on a set A is a ***total relation*** if for all $a, b \in A$, we have $a\ R\ b$ or $b\ R\ a$ (or both), that is, if every pairing of elements of A is related by R. A partial order that is also a total relation is a ***total order*** or ***linear order***.

Let's consider the relation (R) of $\leq$

A total order on $\leq$ is a binary relation that satisfies …

- totality - either $a \leq b$ or $b \leq a$ or both.
- transitivity - if $a \leq b$ and $b \leq c$ then $a \leq c$.
- anti-symmetry - if $a \leq b$ and $b \leq a$ then $a = b$.

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Example: natural numbers:

A total order on ≤ is a binary relation that satisfies …

- totality — $0 \leq 1 \leq 2 \leq 3 \leq 4 \ldots$
- transitivity — $1 \leq 2$ and $2 \leq 3$ so $1 \leq 3$
- anti-symmetry — the only way $a \leq b$ and $b \leq a$ is if $a = c$
  i.e., $1 \leq 1$ and $1 \leq 1$ or $2 \leq 2$ and $2 \leq 2$

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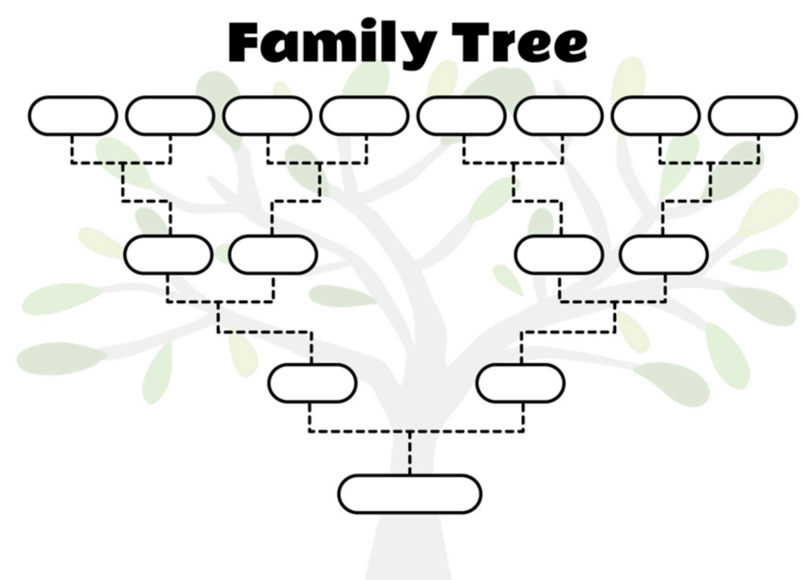
Counter-example: the "is a descendant of" relationship

- totality - Not everybody is related, so this violates totality.



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Counter-example: the "Rock, Paper, Scissors" relationships

- transitivity - scissors < stone and stone < paper, but scissors $\nless$ paper so this violates transitivity. In fact, scissors > paper.

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Counter-example: the "predator, prey" relationships

- anti-symmetry - equal ferocity but different species.
- I.e., we **have** anti-symmetry **unless** $a \leq b$ and $b \leq a$ and $a \neq b$.

Permutations

Order matters

set size	*permu-tations*	*examples*
1	1	{ (a) }
2	2	{ (a,b), (b,a) }
3	6	{ (a,b,c), (a,c,b), (b,a,c), (b,c,a), (c,a,b), (c,b,a) }
4	24	{ (a,b,c,d), (a,b,d,c), . . . }
5	120	{ (a,b,c,d,e), . . . }
.		
.		
.		
?		

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.		
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n!		

Shuffle sort / Bogo sort / Monkey sort

```
procedure sort(in out list D)
begin
   boolean done := false;
   while (not done)
      randomly permute D
      if (D is sorted)
         done := true
      end if
   end while
   // D is returned out
end procedure
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In terms of n, the number of items in list D...

How many times through the loop until we expect it to be sorted?

How long do we expect each iteration to take?

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In terms of n, the number of items in list D...

How many times through the loop until we expect it to be sorted? **n!**

How long do we expect each iteration to take?

permute = $O(n)$
check if sorted = $O(n)$

Total time = time per iteration × number of iterations = $O(n \times n!)$

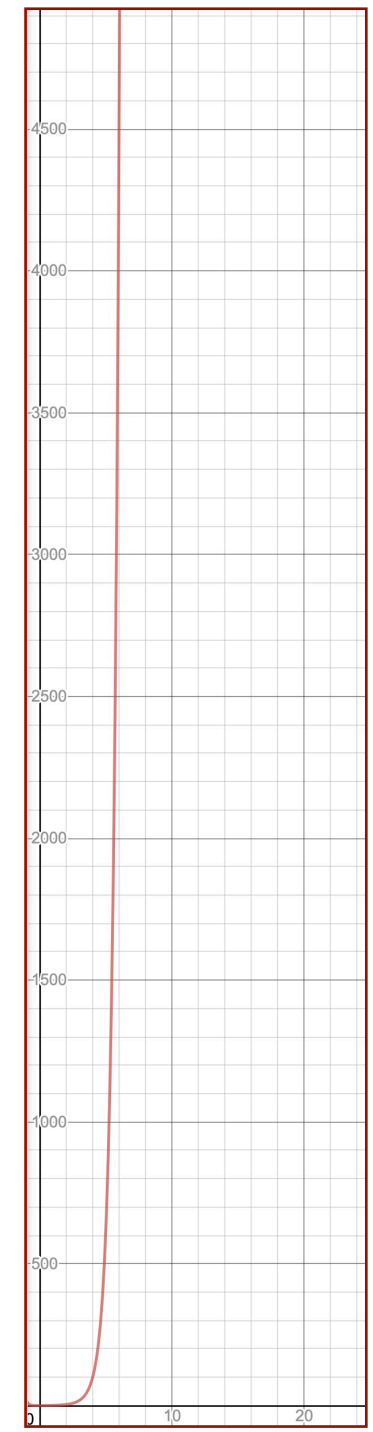
Shuffle sort / Bogo sort / Monkey sort

This is silly. And terrible.

But the worst part is that this is the **expected** case. The worst case scenario is that it never halts because there is no guarantee that we'll ever produce a sorted list through random permutations. In that sense, it's scary.

To put it more technically: O(*scary*)

Let's not do this.



Total time = time per iteration × number of iterations = $O(n \times n!)$

Selection Sort