
Ambiguous Grammars



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Ambiguous Context-Free Grammars

Remember...

- An ambiguous Context-Free Grammar is one where different derivations of the same code can produce different Concrete Syntax Trees.
- This is bad and we'd like to avoid it.
- We'll need to change the grammar to do that.

Ambiguous Context-Free Grammars

Remember this **unambiguous** grammar?

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0|1|2|3|4|5|6|7|8|9$

input tokens: 9 - 5 + 2

Even Earlier slide

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
 2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
 3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
 4. $\langle \text{digit} \rangle ::= 0|1|2|3|4|5|6|7|8|9$

input tokens: X X X X X

5. $\langle \text{list} \rangle$
 1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$
 2. $\langle \text{list} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
 3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
 4. $\boxed{9} \boxed{-} \langle \text{digit} \rangle + \langle \text{digit} \rangle$
 4. $\boxed{9} \boxed{-} \boxed{5} \boxed{+} \langle \text{digit} \rangle$
 4. $\boxed{9} \boxed{-} \boxed{5} \boxed{+} \boxed{2}$

Now what?
 We're good! We've run out of non-terminals to turn into terminals at the same time as we've run out of input tokens to process. This is a beautiful thing... a successful parse.

Earlier slide

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
 2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
 3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
 4. $\langle \text{digit} \rangle ::= 0|1|2|3|4|5|6|7|8|9$

input tokens: X X X X X

5. $\langle \text{list} \rangle$
 1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$
 2. $\langle \text{list} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
 3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
 4. $\boxed{9} \boxed{-} \boxed{5} \boxed{+} \boxed{2}$

We've run out of non-terminals to turn into terminals at the same time as we've run out of input tokens to process. This is a beautiful thing... another successful parse, this time a right-most derivation.

What if we had a grammar a little less cooperative?

Ambiguous Context-Free Grammars

A less cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

Ambiguous Context-Free Grammars

A less cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2



Left-most derivation
 $\langle \text{expr} \rangle$

expr

Ambiguous Context-Free Grammars

A less cooperative grammar

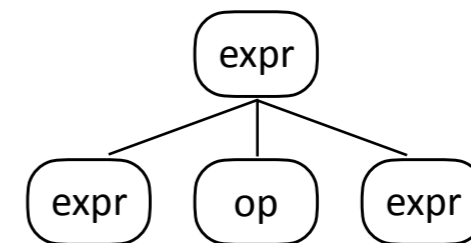
1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

Left-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

input tokens: 8 - 4 ÷ 2



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2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2

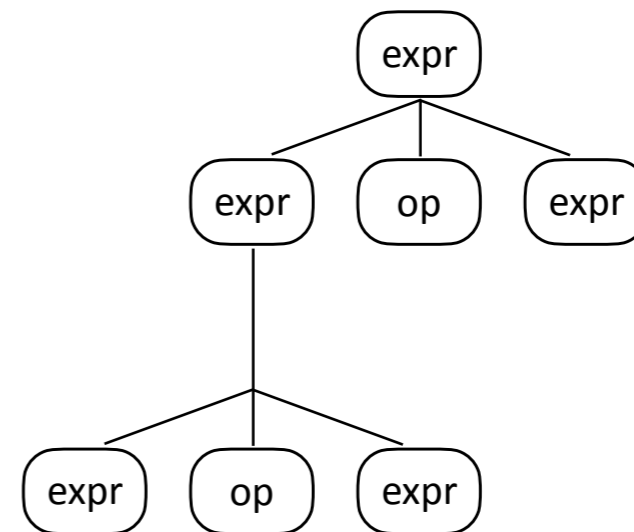


Left-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$



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2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2



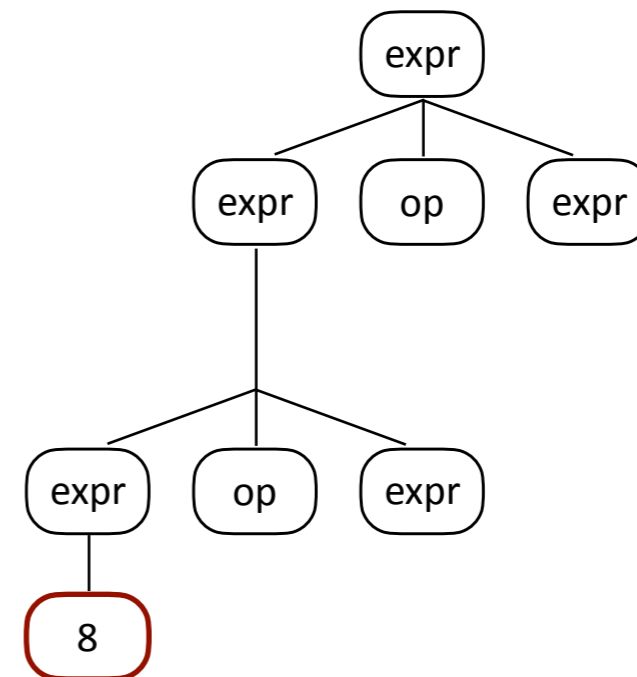
Left-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

[num, 8] $\langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$



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1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2



Left-most derivation

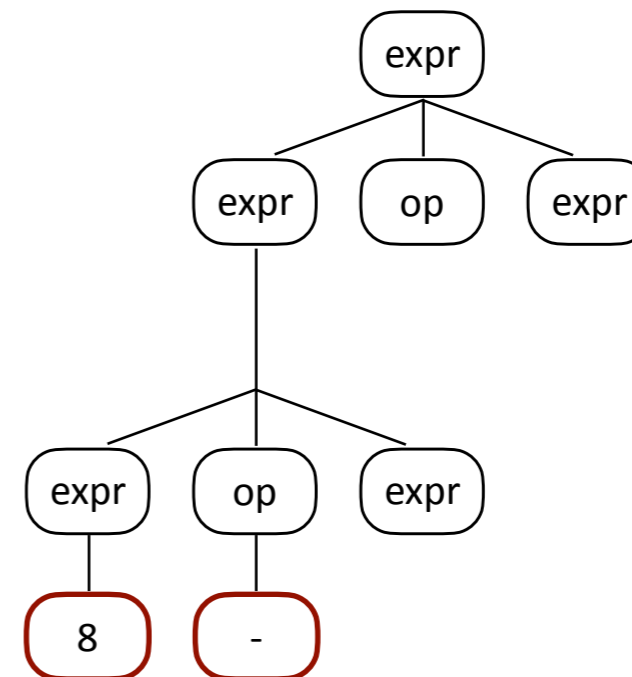
$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$



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A less cooperative grammar

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2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 \div 2

Left-most derivation

$\langle \text{expr} \rangle$

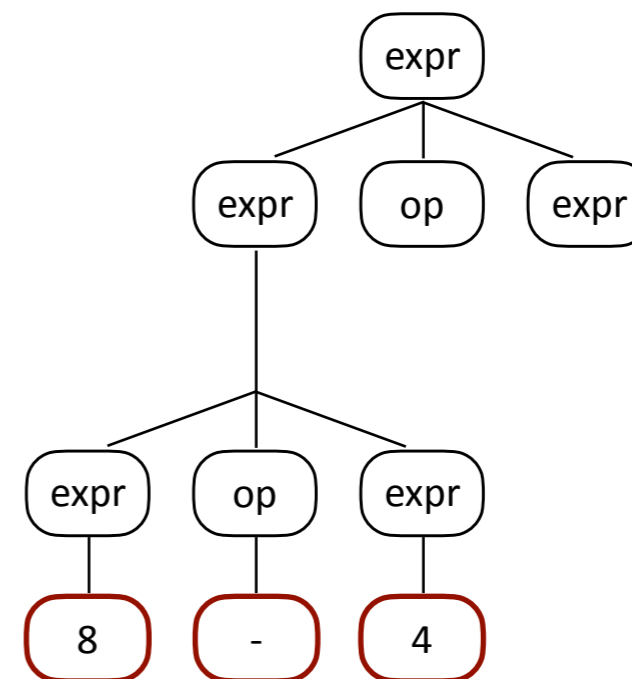
$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \langle \text{op} \rangle \langle \text{expr} \rangle$



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2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2

Left-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

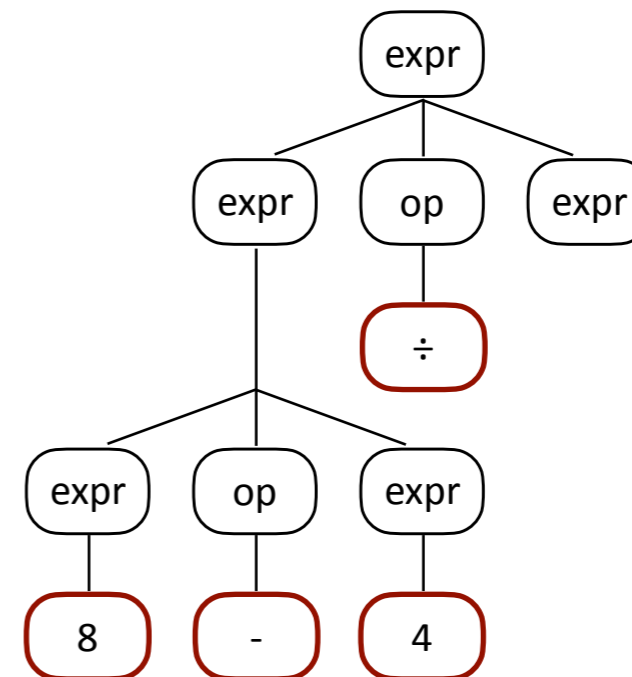
$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \div \langle \text{expr} \rangle$



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3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 \div 2

Left-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

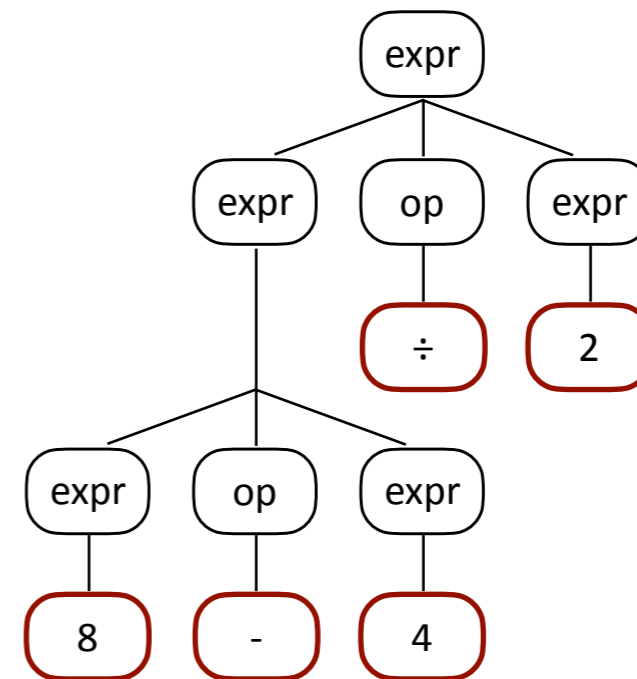
$[\text{num}, 8] \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \div \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \div [\text{num}, 2]$



Ambiguous Context-Free Grammars

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1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2

Left-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

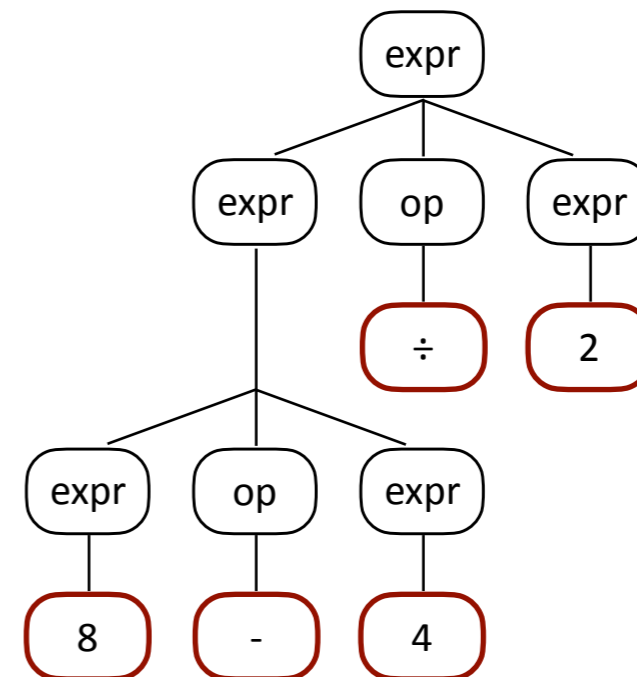
$[\text{num}, 8] \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \div \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \div [\text{num}, 2]$



A depth-first in-order traversal of a CST where we print the leaf nodes as we go will yield 8 - 4 ÷ 2.

The shape of the CST determines the grouping.

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3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2

Left-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

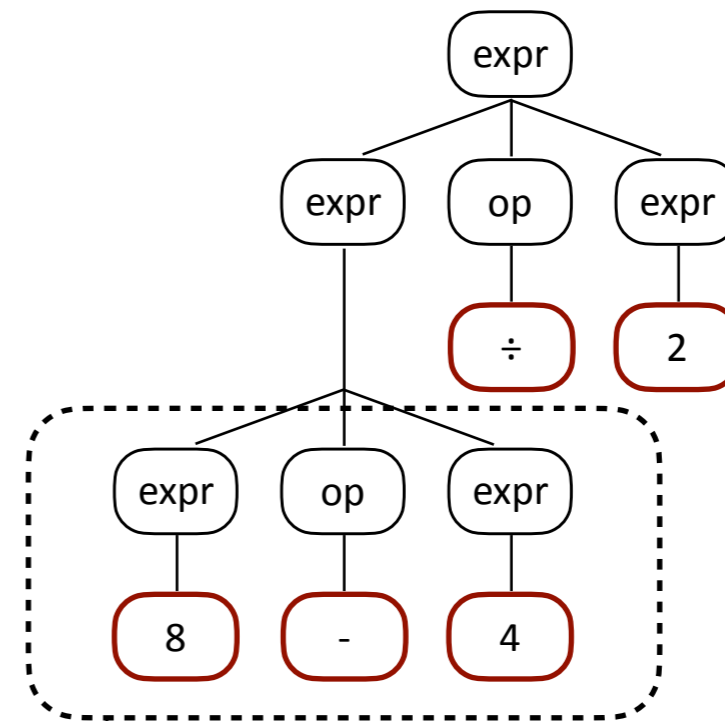
$[\text{num}, 8] \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \div \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \div [\text{num}, 2]$



Because of the shape of the tree: $(8 - 4) \div 2$

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A less cooperative grammar

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2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

Left-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

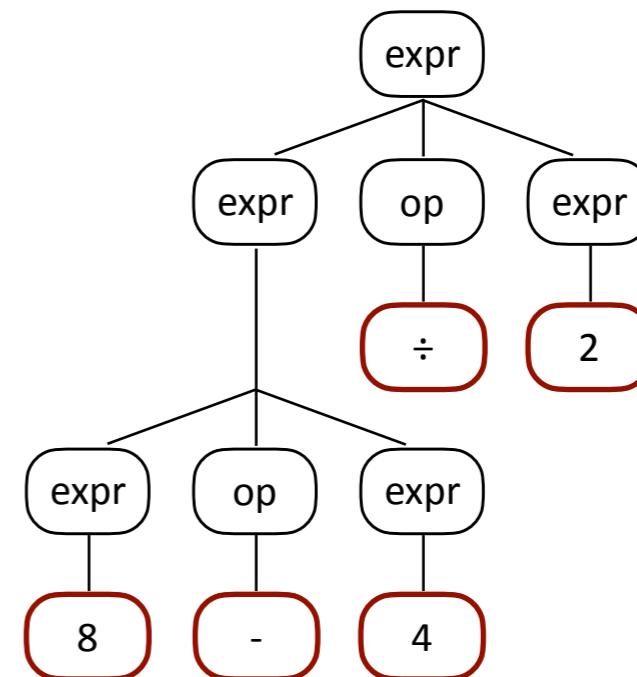
$[\text{num}, 8] - \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \langle \text{op} \rangle \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \div \langle \text{expr} \rangle$

$[\text{num}, 8] - [\text{num}, 4] \div [\text{num}, 2]$

input tokens: 8 - 4 ÷ 2



Because of the shape of the tree: $(8 - 4) \div 2 = 2$, which is wrong.
We broke math.

Ambiguous Context-Free Grammars

A less cooperative grammar

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2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2



Right-most derivation
 $\langle \text{expr} \rangle$

expr

Ambiguous Context-Free Grammars

A less cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
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4. $\quad ::= -$

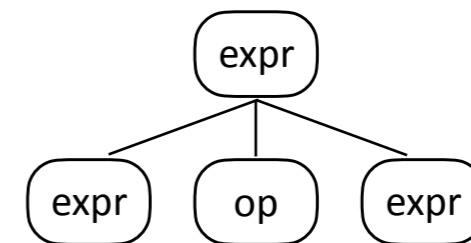
input tokens: 8 - 4 \div 2



Right-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$



Ambiguous Context-Free Grammars

A less cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2

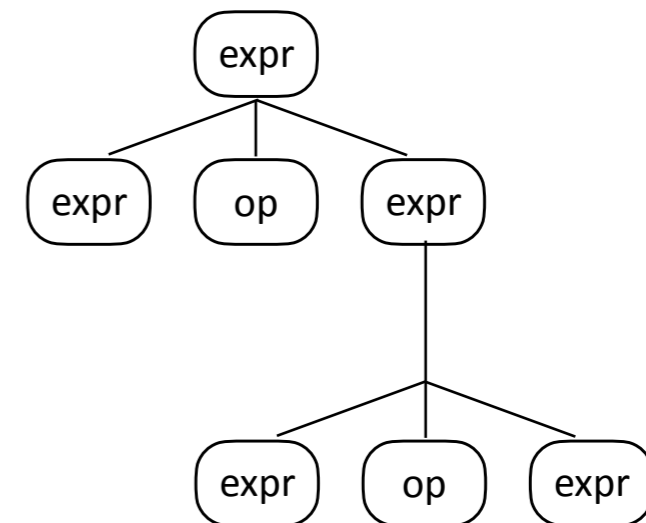


Right-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$



Ambiguous Context-Free Grammars

A less cooperative grammar

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2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 \div 2



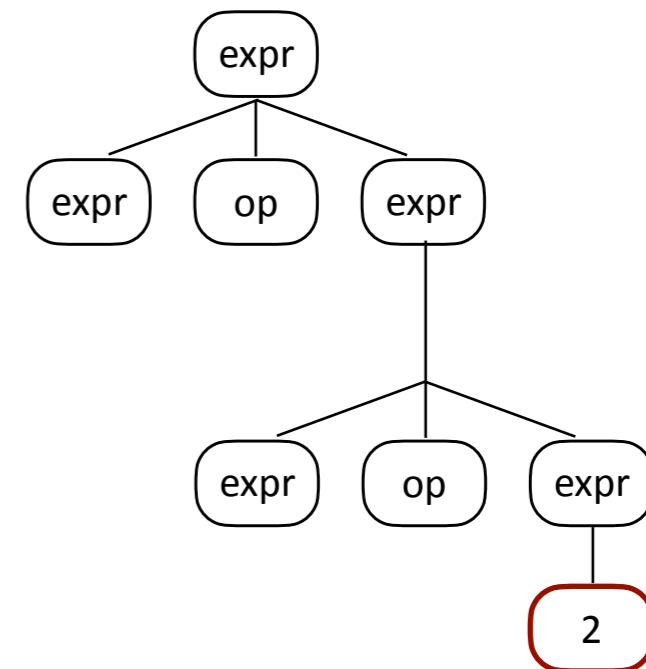
Right-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle$ [num, 2]



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A less cooperative grammar

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2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 \div 2

Right-most derivation

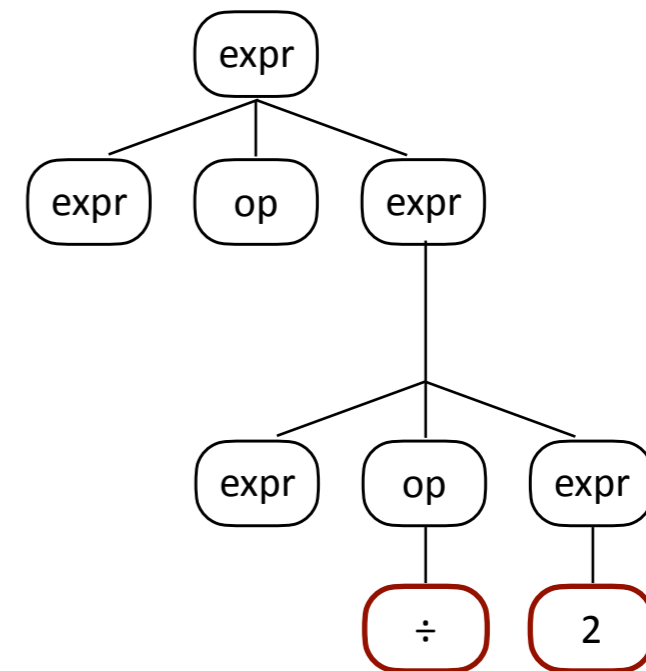
$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle$ [num, 2]

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$ \div [num, 2]



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A less cooperative grammar

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2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2



Right-most derivation

$\langle \text{expr} \rangle$

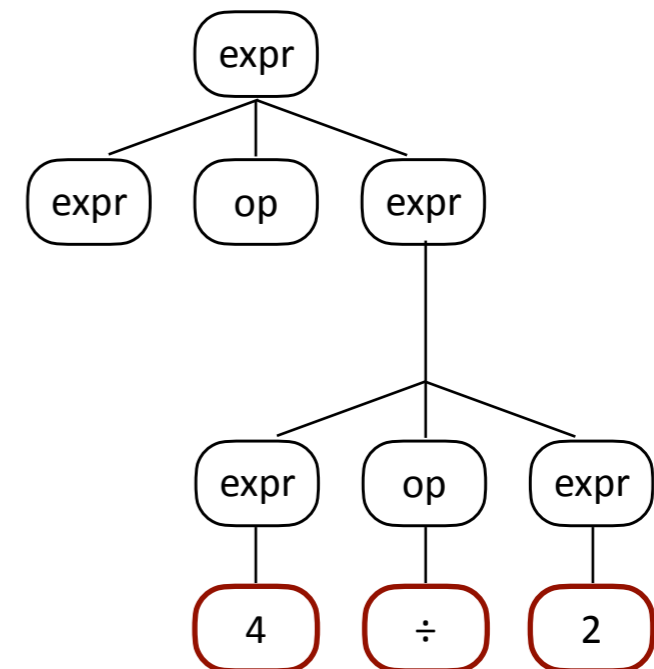
$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle [\text{num}, 2]$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \div [\text{num}, 2]$

$\langle \text{expr} \rangle \langle \text{op} \rangle [\text{num}, 4] \div [\text{num}, 2]$



Ambiguous Context-Free Grammars

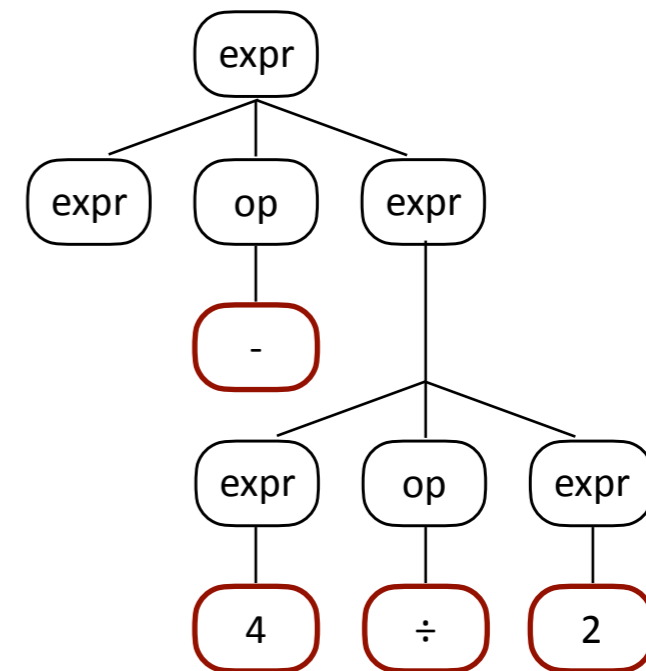
A less cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2

Right-most derivation

$\langle \text{expr} \rangle$
 $\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
 $\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
 $\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle [\text{num}, 2]$
 $\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \div [\text{num}, 2]$
 $\langle \text{expr} \rangle \langle \text{op} \rangle [\text{num}, 4] \div [\text{num}, 2]$
 $\langle \text{expr} \rangle - [\text{num}, 4] \div [\text{num}, 2]$



Ambiguous Context-Free Grammars

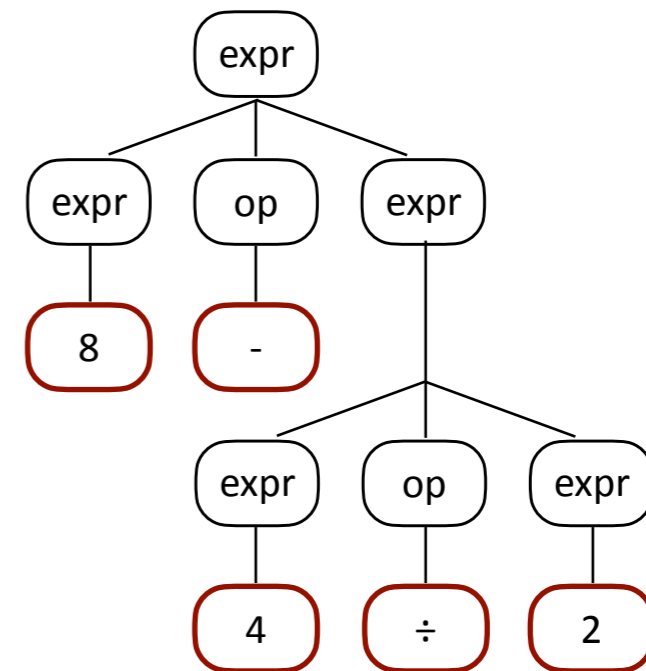
A less cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2

Right-most derivation

$\langle \text{expr} \rangle$				
$\langle \text{expr} \rangle$	$\langle \text{op} \rangle$	$\langle \text{expr} \rangle$		
$\langle \text{expr} \rangle$	$\langle \text{op} \rangle$	$\langle \text{expr} \rangle$	$\langle \text{op} \rangle$	$\langle \text{expr} \rangle$
$\langle \text{expr} \rangle$	$\langle \text{op} \rangle$	$\langle \text{expr} \rangle$	$\langle \text{op} \rangle$	$[\text{num}, 2]$
$\langle \text{expr} \rangle$	$\langle \text{op} \rangle$	$\langle \text{expr} \rangle$	\div	$[\text{num}, 2]$
$\langle \text{expr} \rangle$	$\langle \text{op} \rangle$	$[\text{num}, 4]$	\div	$[\text{num}, 2]$
$\langle \text{expr} \rangle$	$-$	$[\text{num}, 4]$	\div	$[\text{num}, 2]$
$[\text{num}, 8]$	$-$	$[\text{num}, 4]$	\div	$[\text{num}, 2]$



Ambiguous Context-Free Grammars

A less cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2

Right-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

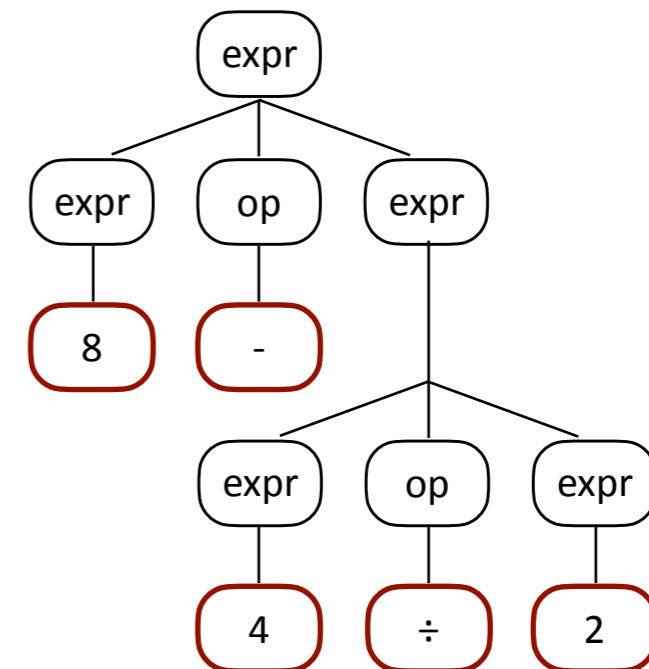
$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle [\text{num}, 2]$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \div [\text{num}, 2]$

$\langle \text{expr} \rangle \langle \text{op} \rangle [\text{num}, 4] \div [\text{num}, 2]$

$\langle \text{expr} \rangle - [\text{num}, 4] \div [\text{num}, 2]$

$[\text{num}, 8] - [\text{num}, 4] \div [\text{num}, 2]$



A depth-first in-order traversal of a CST where we print the leaf nodes as we go will yield 8 - 4 ÷ 2.

The shape of the CST determines the grouping.

Ambiguous Context-Free Grammars

A less cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2

Right-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

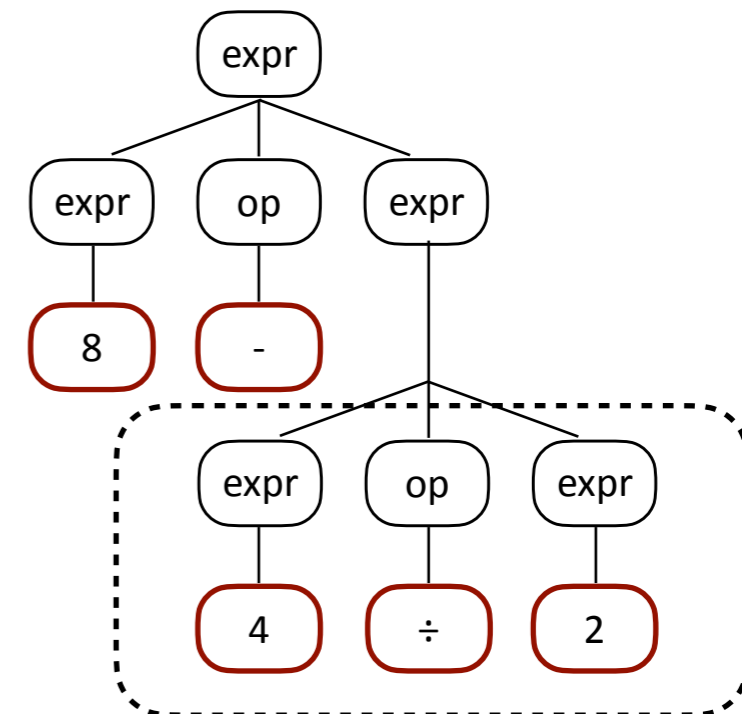
$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle [\text{num}, 2]$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \div [\text{num}, 2]$

$\langle \text{expr} \rangle \langle \text{op} \rangle [\text{num}, 4] \div [\text{num}, 2]$

$\langle \text{expr} \rangle - [\text{num}, 4] \div [\text{num}, 2]$

$[\text{num}, 8] - [\text{num}, 4] \div [\text{num}, 2]$



Because of the shape of the tree: 8 - (4 ÷ 2)

Ambiguous Context-Free Grammars

A less cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
2. $\quad ::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $\quad ::= -$

input tokens: 8 - 4 ÷ 2

Right-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$

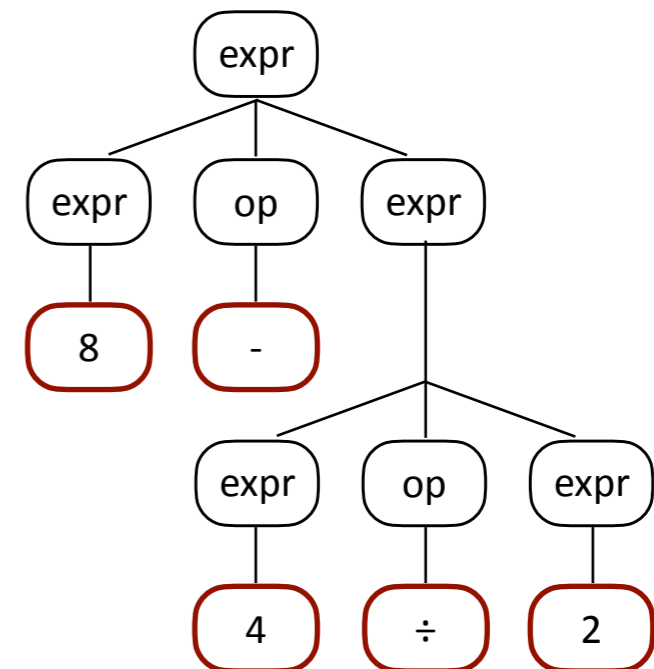
$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \langle \text{op} \rangle [\text{num}, 2]$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \div [\text{num}, 2]$

$\langle \text{expr} \rangle \langle \text{op} \rangle [\text{num}, 4] \div [\text{num}, 2]$

$\langle \text{expr} \rangle - [\text{num}, 4] \div [\text{num}, 2]$

$[\text{num}, 8] - [\text{num}, 4] \div [\text{num}, 2]$



Because of the shape of the tree: $8 - (4 \div 2) = 6$, which is correct.
Yay, math!

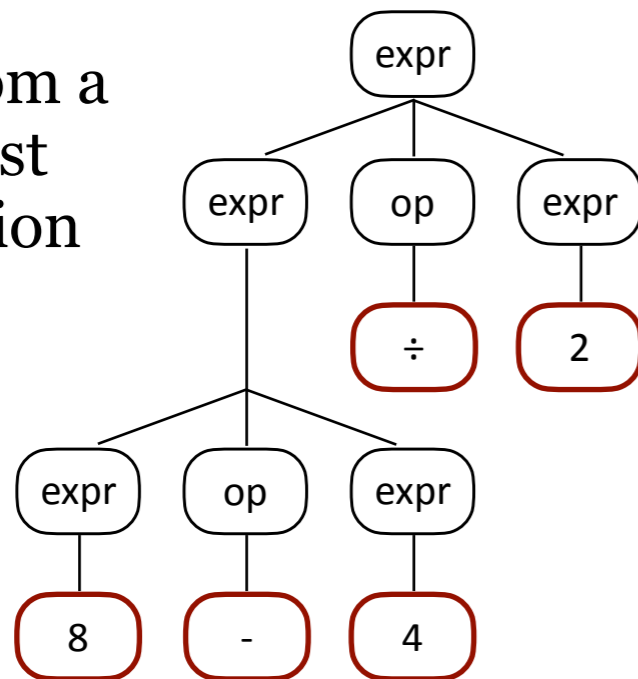
Ambiguous Context-Free Grammars

A less cooperative grammar

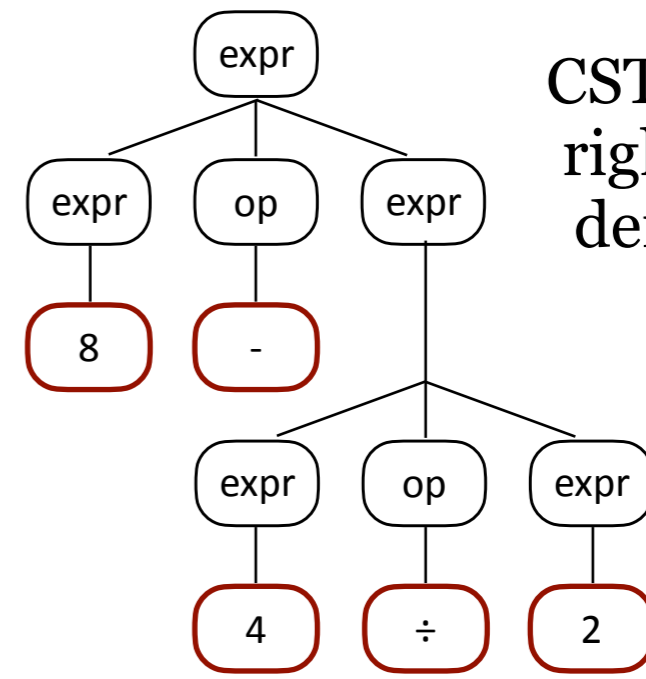
1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$
2. $::= \text{num}$
3. $\langle \text{op} \rangle ::= \div$
4. $::= -$

input tokens: 8 - 4 ÷ 2

CST from a
left-most
derivation



CST from a
right-most
derivation



This is what happens with ambiguous grammars. Different derivations can lead to different parse trees, which leads to different meanings of the program, which leads to suffering.

Ambiguous Context-Free Grammars

A more cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle - \langle \text{term} \rangle$
2. $\langle \text{expr} \rangle ::= \langle \text{term} \rangle$
3. $\langle \text{term} \rangle ::= \langle \text{term} \rangle \div \langle \text{num} \rangle$
4. $\langle \text{term} \rangle ::= \langle \text{num} \rangle$
5. $\langle \text{num} \rangle ::= 0 \mid 1 \mid 2 \mid \dots \mid 9$

Context-Free Grammars

A more cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle - \langle \text{term} \rangle$
2. $\langle \text{expr} \rangle ::= \langle \text{term} \rangle$
3. $\langle \text{term} \rangle ::= \langle \text{term} \rangle \div \langle \text{num} \rangle$
4. $\langle \text{term} \rangle ::= \langle \text{num} \rangle$
5. $\langle \text{num} \rangle ::= 0 \mid 1 \mid 2 \mid \dots \mid 9$

Left-most derivation

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle - \langle \text{term} \rangle$

$\langle \text{term} \rangle - \langle \text{term} \rangle$

$\langle \text{num} \rangle - \langle \text{term} \rangle$

8 - $\langle \text{term} \rangle$

8 - $\langle \text{term} \rangle \div \langle \text{num} \rangle$

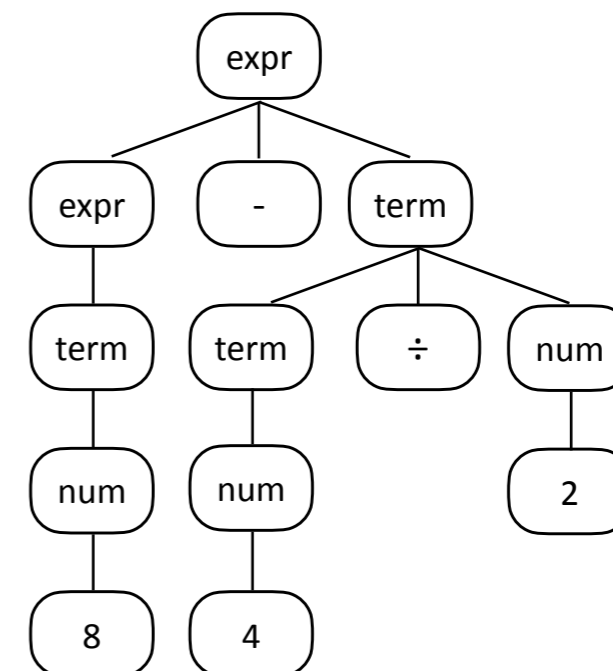
8 - $\langle \text{num} \rangle \div \langle \text{num} \rangle$

8 - **4** $\div \langle \text{num} \rangle$

8 - **4** $\div \langle \text{num} \rangle$

8 - **4** \div **2**

input tokens: 8 - 4 ÷ 2



Context-Free Grammars

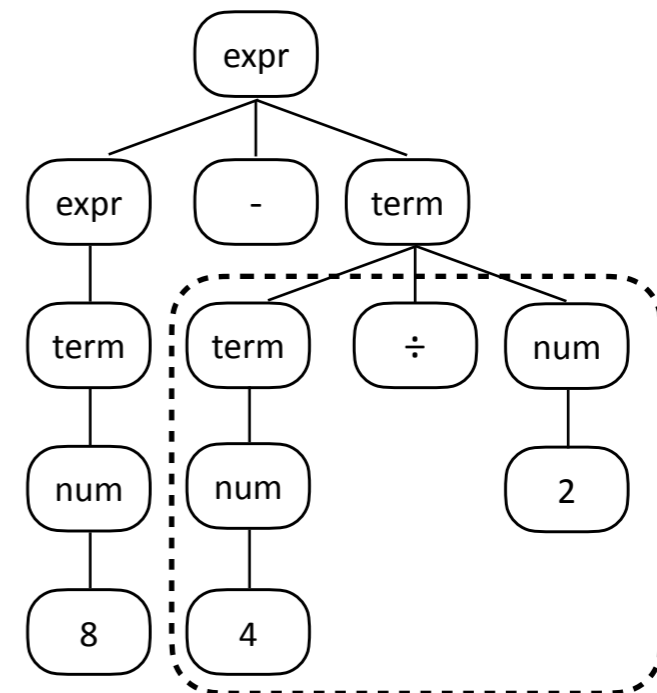
A more cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle - \langle \text{term} \rangle$
2. $\langle \text{expr} \rangle ::= \langle \text{term} \rangle$
3. $\langle \text{term} \rangle ::= \langle \text{term} \rangle \div \langle \text{num} \rangle$
4. $\langle \text{term} \rangle ::= \langle \text{num} \rangle$
5. $\langle \text{num} \rangle ::= 0 \mid 1 \mid 2 \cdots \mid 9$

Left-most derivation

$\langle \text{expr} \rangle$
 $\langle \text{expr} \rangle - \langle \text{term} \rangle$
 $\langle \text{term} \rangle - \langle \text{term} \rangle$
 $\langle \text{num} \rangle - \langle \text{term} \rangle$
8 - $\langle \text{term} \rangle$
8 - $\langle \text{term} \rangle \div \langle \text{num} \rangle$
8 - $\langle \text{num} \rangle \div \langle \text{num} \rangle$
8 - **4** $\div \langle \text{num} \rangle$
8 - **4** $\div \langle \text{num} \rangle$
8 - **4** \div **2**

input tokens: 8 - 4 ÷ 2



output: 8 - (4 ÷ 2)

Context-Free Grammars

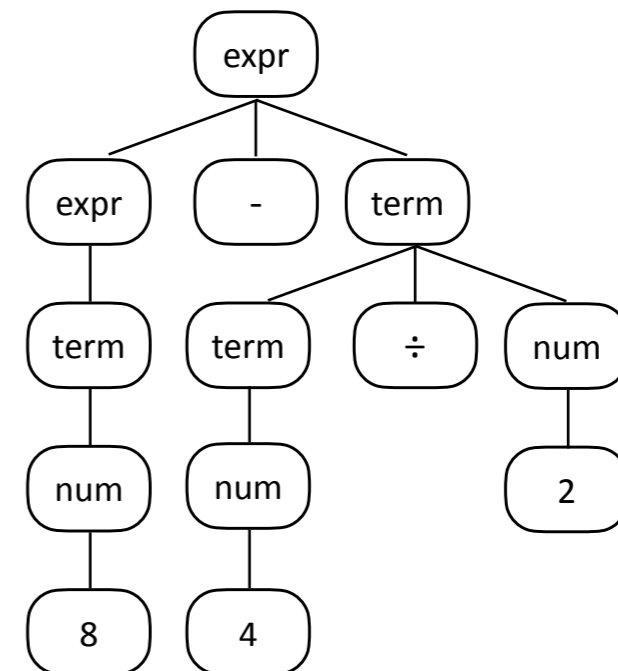
A more cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle - \langle \text{term} \rangle$
2. $\quad ::= \langle \text{term} \rangle$
3. $\langle \text{term} \rangle ::= \langle \text{term} \rangle \div \langle \text{num} \rangle$
4. $\quad ::= \langle \text{num} \rangle$
5. $\langle \text{num} \rangle ::= 0 \mid 1 \mid 2 \cdots \mid 9$

Right-most derivation

$\langle \text{expr} \rangle$
 $\langle \text{expr} \rangle - \langle \text{term} \rangle$
 $\langle \text{expr} \rangle - \langle \text{term} \rangle \div \langle \text{num} \rangle$
 $\langle \text{expr} \rangle - \langle \text{term} \rangle \div 2$
 $\langle \text{expr} \rangle - \langle \text{term} \rangle \div 2$
 $\langle \text{expr} \rangle - \langle \text{num} \rangle \div 2$
 $\langle \text{expr} \rangle - 4 \div 2$
 $\langle \text{expr} \rangle - 4 \div 2$
 $\langle \text{term} \rangle - 4 \div 2$
 $\langle \text{num} \rangle - 4 \div 2$
 $8 - 4 \div 2$

input tokens: 8 - 4 ÷ 2



Context-Free Grammars

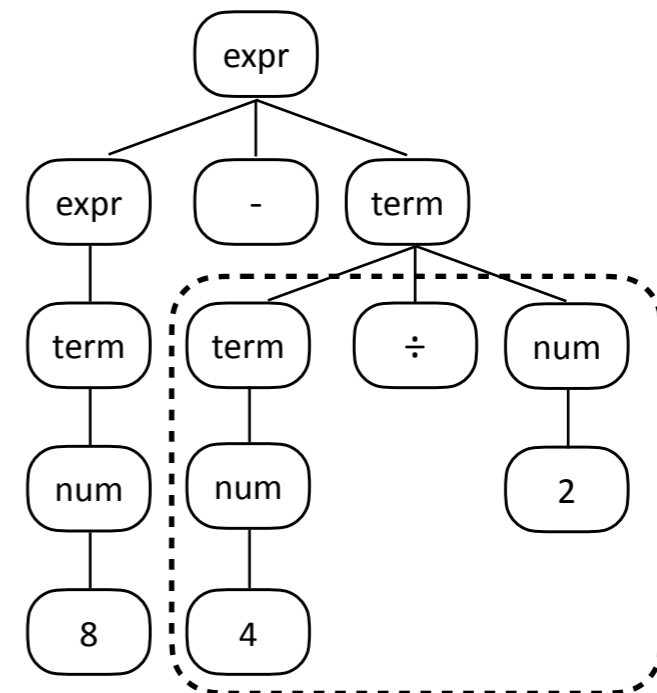
A more cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle - \langle \text{term} \rangle$
2. $\langle \text{expr} \rangle ::= \langle \text{term} \rangle$
3. $\langle \text{term} \rangle ::= \langle \text{term} \rangle \div \langle \text{num} \rangle$
4. $\langle \text{term} \rangle ::= \langle \text{num} \rangle$
5. $\langle \text{num} \rangle ::= 0 \mid 1 \mid 2 \cdots \mid 9$

Right-most derivation

$\langle \text{expr} \rangle$
 $\langle \text{expr} \rangle - \langle \text{term} \rangle$
 $\langle \text{expr} \rangle - \langle \text{term} \rangle \div \langle \text{num} \rangle$
 $\langle \text{expr} \rangle - \langle \text{term} \rangle \div 2$
 $\langle \text{expr} \rangle - \langle \text{term} \rangle \div 2$
 $\langle \text{expr} \rangle - \langle \text{num} \rangle \div 2$
 $\langle \text{expr} \rangle - 4 \div 2$
 $\langle \text{expr} \rangle - 4 \div 2$
 $\langle \text{term} \rangle - 4 \div 2$
 $\langle \text{num} \rangle - 4 \div 2$
 $8 - 4 \div 2$

input tokens: 8 - 4 ÷ 2



output: 8 - (4 ÷ 2)

Context-Free Grammars

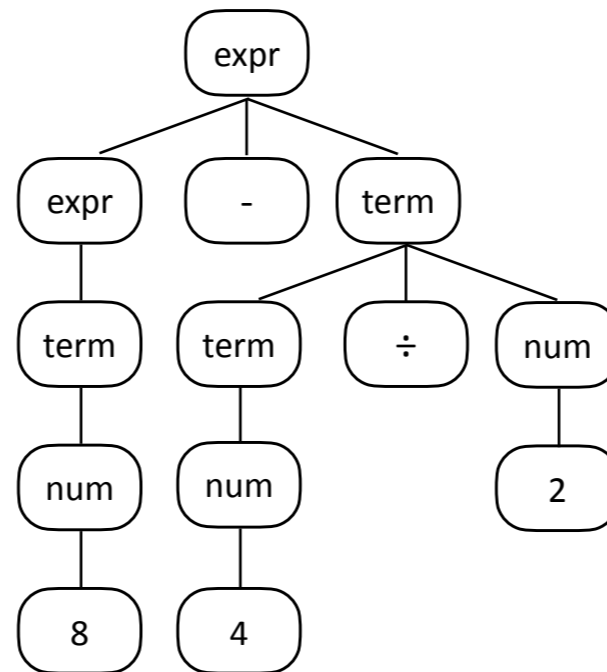
A very cooperative grammar

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle - \langle \text{term} \rangle$
2. $\quad ::= \langle \text{term} \rangle$
3. $\langle \text{term} \rangle ::= \langle \text{term} \rangle \div \langle \text{num} \rangle$
4. $\quad ::= \langle \text{num} \rangle$
5. $\langle \text{num} \rangle ::= 0 \mid 1 \mid 2 \cdots \mid 9$

input tokens: 8 - 4 ÷ 2

Left-most derivation

$\langle \text{expr} \rangle$			
$\langle \text{expr} \rangle$	-	$\langle \text{term} \rangle$	
$\langle \text{term} \rangle$	-	$\langle \text{term} \rangle$	
$\langle \text{num} \rangle$	-	$\langle \text{term} \rangle$	
8	-	$\langle \text{term} \rangle$	
8	-	$\langle \text{term} \rangle \div \langle \text{num} \rangle$	
8	-	$\langle \text{num} \rangle \div \langle \text{num} \rangle$	
8	-	4 ÷ $\langle \text{num} \rangle$	
8	-	4 ÷ $\langle \text{num} \rangle$	
8	-	4 ÷ 2	



Right-most derivation

$\langle \text{expr} \rangle$			
$\langle \text{expr} \rangle$	-	$\langle \text{term} \rangle$	
$\langle \text{expr} \rangle$	-	$\langle \text{term} \rangle \div \langle \text{num} \rangle$	
$\langle \text{expr} \rangle$	-	$\langle \text{term} \rangle \div 2$	
$\langle \text{expr} \rangle$	-	$\langle \text{term} \rangle \div 2$	
$\langle \text{expr} \rangle$	-	$\langle \text{num} \rangle \div 2$	
$\langle \text{expr} \rangle$	-	4 ÷ 2	
$\langle \text{expr} \rangle$	-	4 ÷ 2	
$\langle \text{term} \rangle$	-	4 ÷ 2	
$\langle \text{num} \rangle$	-	4 ÷ 2	
8	-	4 ÷ 2	

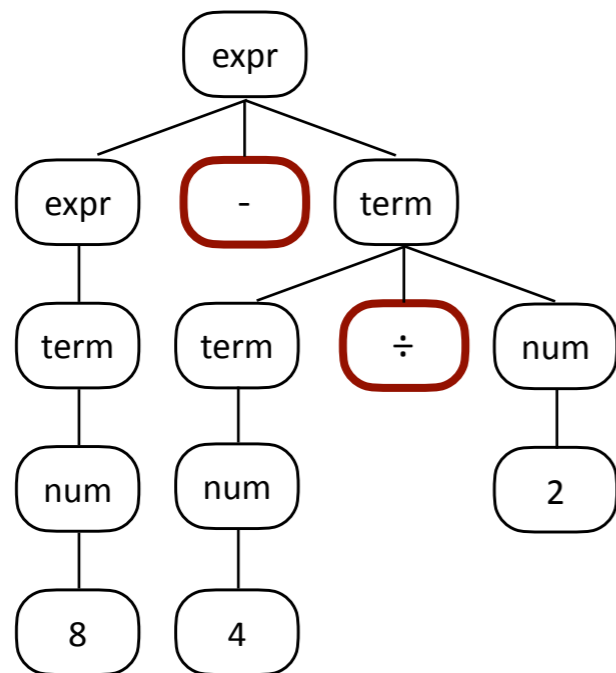
Same CST because
the grammar is unambiguous.

Context-Free Grammars

A note about operator precedence.

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle - \langle \text{term} \rangle$
2. $\langle \text{term} \rangle ::= \langle \text{term} \rangle \div \langle \text{num} \rangle$
3. $\langle \text{term} \rangle ::= \langle \text{num} \rangle$
4. $\langle \text{num} \rangle ::= 0 \mid 1 \mid 2 \mid \dots \mid 9$

input tokens: 8 - 4 ÷ 2



Note:

The **higher** the operator precedence, the **lower** it goes in the syntax tree.

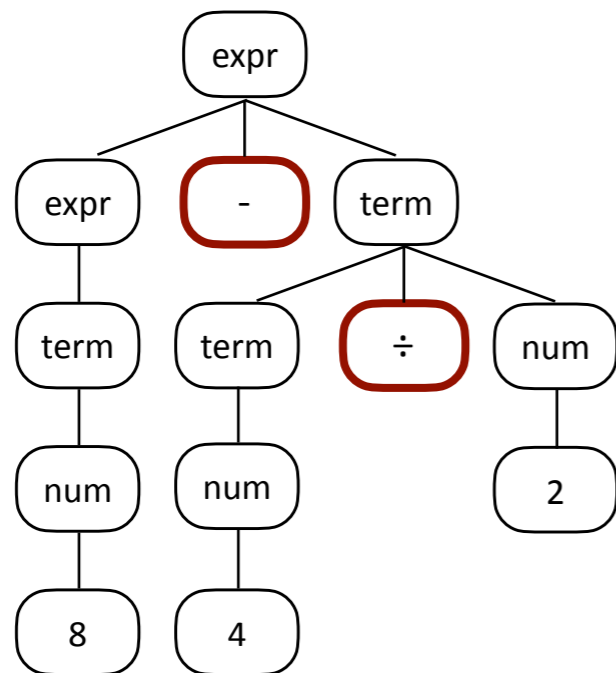
Why is this?

Context-Free Grammars

A note about operator precedence.

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle - \langle \text{term} \rangle$
2. $\langle \text{term} \rangle ::= \langle \text{term} \rangle \div \langle \text{num} \rangle$
3. $\langle \text{term} \rangle ::= \langle \text{num} \rangle$
4. $\langle \text{num} \rangle ::= 0 \mid 1 \mid 2 \cdots \mid 9$

input tokens: 8 - 4 ÷ 2



Note:

The **higher** the operator precedence, the **lower** it goes in the syntax tree.

This is because of the grammar.

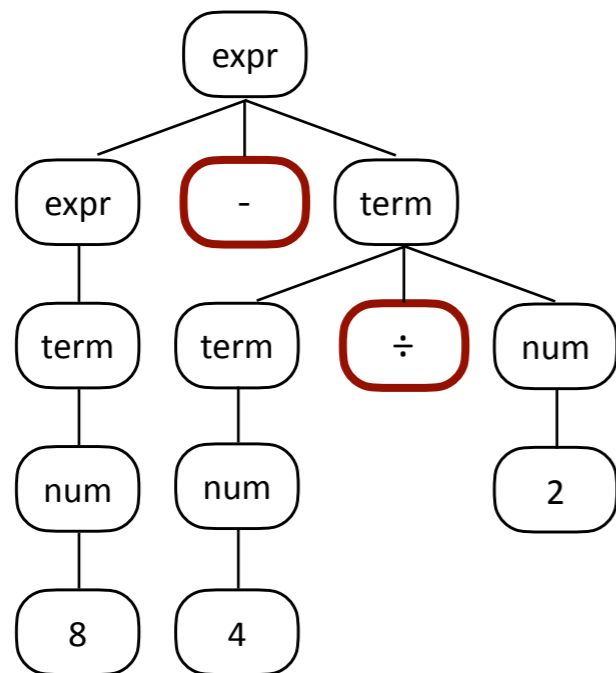
In this example, if there is subtraction, it has to be processed before division because we can only divide $\langle \text{term} \rangle$ s so we'll have to go through production #1 to get there, thus placing minus higher in the derivation — and therefore higher in the syntax tree — than division.

Context-Free Grammars

A note about operator precedence.

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle - \langle \text{term} \rangle$
2. $\langle \text{term} \rangle ::= \langle \text{term} \rangle \div \langle \text{num} \rangle$
3. $\langle \text{term} \rangle ::= \langle \text{num} \rangle$
4. $\langle \text{num} \rangle ::= 0 \mid 1 \mid 2 \mid \dots \mid 9$

input tokens: 8 - 4 ÷ 2



Note:

The **higher** the operator precedence, the **lower** it goes in the syntax tree.

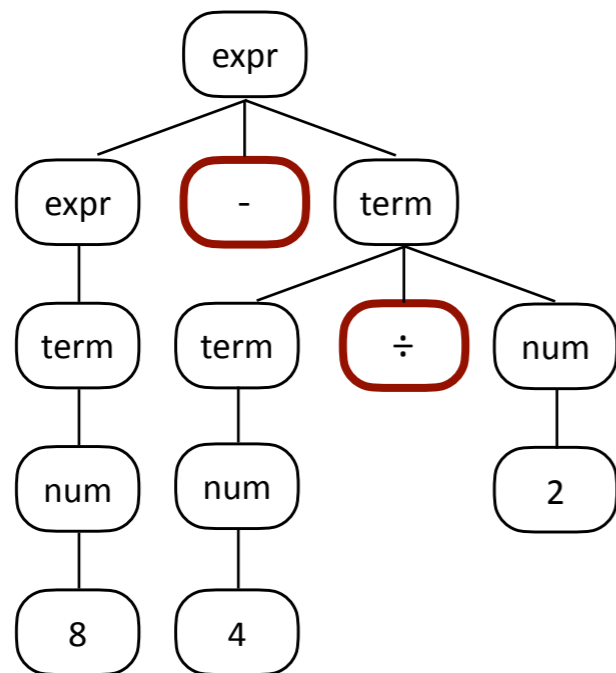
Why do we want this?

Context-Free Grammars

A note about operator precedence.

1. $\langle \text{expr} \rangle ::= \langle \text{expr} \rangle - \langle \text{term} \rangle$
2. $\langle \text{term} \rangle ::= \langle \text{term} \rangle \div \langle \text{num} \rangle$
3. $\langle \text{num} \rangle ::= 0 \mid 1 \mid 2 \mid \dots \mid 9$

input tokens: 8 - 4 ÷ 2



Note:

The **higher** the operator precedence, the **lower** it goes in the syntax tree.

We want this because we're going to process our CSTs with depth-first in-order traversals. Since it's **depth-first**, we'll get to and process operators **lower** on the tree before the we get to and process operators higher up the tree.