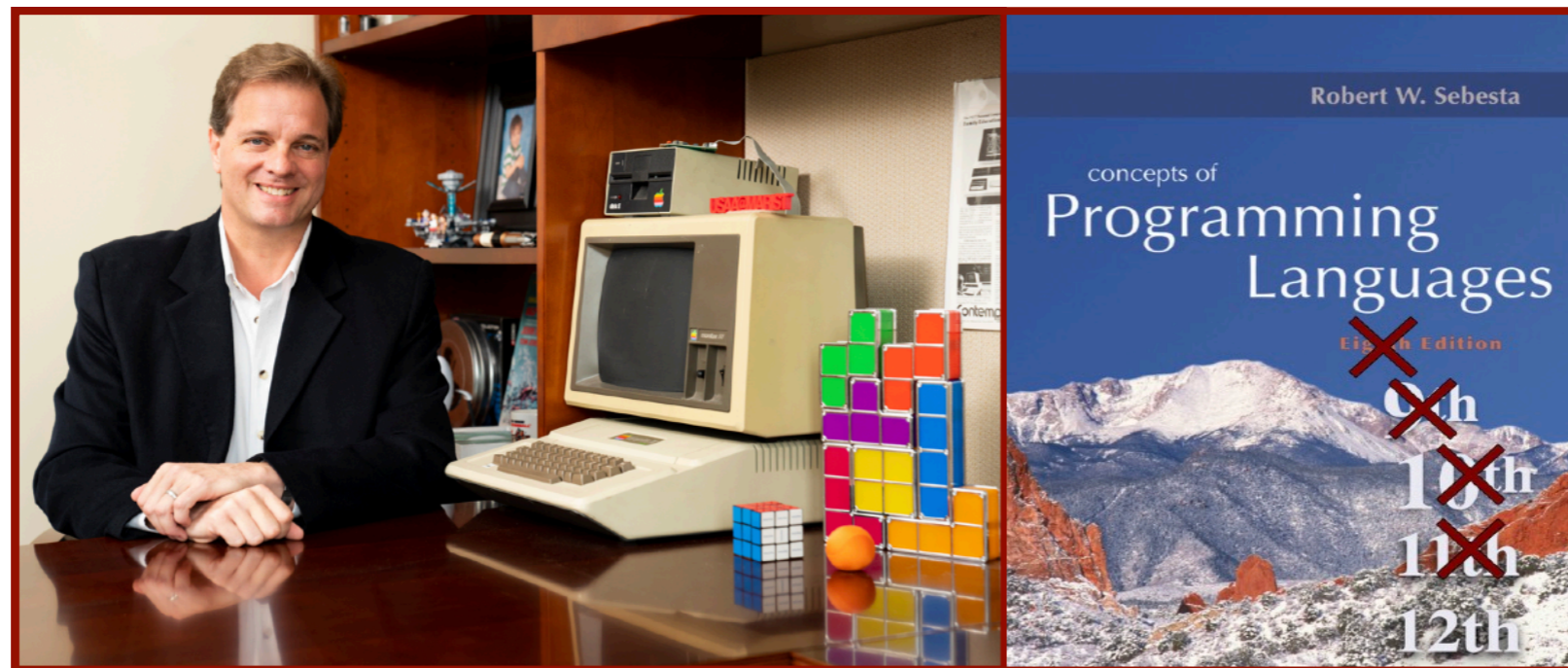

Context-Free Grammars



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Context-Free Grammars

Defining a context-free grammar, G :

$$G = (S, N, T, P)$$

where

S is the start symbol

N is a set of non-terminal symbols

T is a set of terminal symbols

P is a set of productions (or *rewrite rules*)

Backus-Naur Form (BNF) is a context-free grammar that's used to define context-free grammars (including itself).

It's recursively cool!

Context-Free Grammars

Backus-Naur Form (BNF)

Consider a grammar for the noise a sheep makes.

1. `<sheep noise> → baa`

Context-Free Grammars

Backus-Naur Form (BNF)

Consider a grammar for the noise a sheep makes.

1. $\langle \text{sheep noise} \rangle \rightarrow \text{baa}$

S is the start symbol.

N is a set of non-terminal symbols.

T is a set of terminal symbols.

P is a set of productions.

Context-Free Grammars

Backus-Naur Form (BNF)

Consider a grammar for the noise a sheep makes.

1. `<sheep noise>` \rightarrow `baa`

$$S = \langle \text{sheep noise} \rangle$$
$$N = \{ \langle \text{sheep noise} \rangle \}$$
$$T = \{ \text{baa} \}$$
$$P = \{ 1 \}$$

If the input is `baa` we can prove that it's a valid sentence in our grammar by showing each step of the parse.

Context-Free Grammars

Backus-Naur Form (BNF)

Consider a grammar for the noise a sheep makes.

1. `<sheep noise> → baa`

input token: `baa`

Parse:

By production #1 we can replace `<sheep noise>` with `baa`. That consumes the first (and only) token from the input. We've exhausted the input tokens and there are no non-terminals to be replaced with terminals so we're done and have a successful parse.

Easy.

But this only supports very terse sheep. What about `baa baa` ?

Context-Free Grammars

Backus-Naur Form (BNF)

Consider a grammar for the noise a sheep makes.

1. `<sheep noise> → baa`

input tokens: `baa baa`

Parse:

By production #1 we can replace `<sheep noise>` with `baa`. That consumes the first token from the input. We've **not** exhausted the input tokens but there are no non-terminals to be replaced with terminals so we're done and have an **unsuccessful** parse, indicating an error.

We need another production.

Context-Free Grammars

Backus-Naur Form (BNF)

Consider a grammar for the noise a sheep makes.

1. `<sheep noise>` \rightarrow `baa`
2. `<sheep noise>` \rightarrow `baa <sheep noise>`

input tokens: `baa baa`

Parse:

By production #2 we can replace `<sheep noise>` with `baa <sheep noise>`. That consumes the first token from the input.

By production #1 we can replace `<sheep noise>` with `baa`.

That consumes the second and last token from the input.

We've exhausted the input tokens and there are no more non-terminals to be replaced with terminals so we're done and have a successful parse.

Context-Free Grammars

Backus-Naur Form (BNF)

Consider a grammar for the noise a sheep makes.

1. `<sheep noise>` \rightarrow `baa`

2. `<sheep noise>` \rightarrow `baa <sheep noise>`

Recursion!

input tokens: `baa baa`

Parse:

By production #2 we can replace `<sheep noise>` with `baa <sheep noise>`. That consumes the first token from the input.

By production #1 we can replace `<sheep noise>` with `baa`.

That consumes the second and last token from the input.

We've exhausted the input tokens and there are no more non-terminals to be replaced with terminals so we're done and have a successful parse.

Context-Free Grammars

1. $\langle \text{goal} \rangle \rightarrow \langle \text{expr} \rangle$
2. $\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{term} \rangle$
3. $\quad \quad \quad \rightarrow \langle \text{term} \rangle$
4. $\langle \text{term} \rangle \rightarrow \text{num}$
5. $\quad \quad \quad \rightarrow \text{id}$
6. $\langle \text{op} \rangle \quad \rightarrow +$
7. $\quad \quad \quad \rightarrow -$

$S = \langle \text{goal} \rangle$

$N = \{ \langle \text{goal} \rangle, \langle \text{expr} \rangle, \langle \text{term} \rangle, \langle \text{op} \rangle \}$

$T = \{ \text{num}, \text{id}, +, - \}$

$P = \{ 1, 2, 3, 4, 5, 6, 7 \}$

Context-Free Grammars

1. $\langle \text{goal} \rangle \rightarrow \langle \text{expr} \rangle$
2. $\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{term} \rangle$
3. $\langle \text{expr} \rangle \rightarrow \langle \text{term} \rangle$
4. $\langle \text{term} \rangle \rightarrow \text{num}$
5. $\langle \text{term} \rangle \rightarrow \text{id}$
6. $\langle \text{op} \rangle \rightarrow +$
7. $\langle \text{op} \rangle \rightarrow -$

input tokens: $x + 2 - y$

Left-most derivation

$\langle \text{goal} \rangle$

$\langle \text{expr} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{term} \rangle$

$\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{term} \rangle \langle \text{op} \rangle \langle \text{term} \rangle$

$\langle \text{term} \rangle \langle \text{op} \rangle \langle \text{term} \rangle \langle \text{op} \rangle \langle \text{term} \rangle$

$[id, x] \langle \text{op} \rangle \langle \text{term} \rangle \langle \text{op} \rangle \langle \text{term} \rangle$

$[id, x] + \langle \text{term} \rangle \langle \text{op} \rangle \langle \text{term} \rangle$

$[id, x] + [num, 2] \langle \text{op} \rangle \langle \text{term} \rangle$

$[id, x] + [num, 2] - \langle \text{term} \rangle$

$[id, x] + [num, 2] - [id, y]$

Context-Free Grammars

1. $\langle \text{goal} \rangle \rightarrow \langle \text{expr} \rangle$
2. $\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{term} \rangle$
3. $\quad \quad \quad \rightarrow \langle \text{term} \rangle$
4. $\langle \text{term} \rangle \rightarrow \text{num}$
5. $\quad \quad \quad \rightarrow \text{id}$
6. $\langle \text{op} \rangle \rightarrow +$
7. $\quad \quad \quad \rightarrow -$

input tokens: $x + 2 - y$

Left-most derivation

```

<goal>
<expr>
<expr> <op> <term>
<expr> <op> <term> <op> <term>
<term> <op> <term> <op> <term>
[id,x] <op> <term> <op> <term>
[id,x] + <term> <op> <term>
[id,x] + [num,2] <op> <term>
[id,x] + [num,2] - <term>
[id,x] + [num,2] - [id,y]
  
```

Right-most derivation

```

<goal>
<expr>
<expr> <op> <term>
<expr> <op> [id,y]
<expr> - [id,y]
<expr> <op> <term> - [id,y]
<expr> <op> [num,2] - [id,y]
<expr> + [num,2] - [id,y]
<term> + [num,2] - [id,y]
[id,x] + [num,2] - [id,y]
  
```

Context-Free Grammars

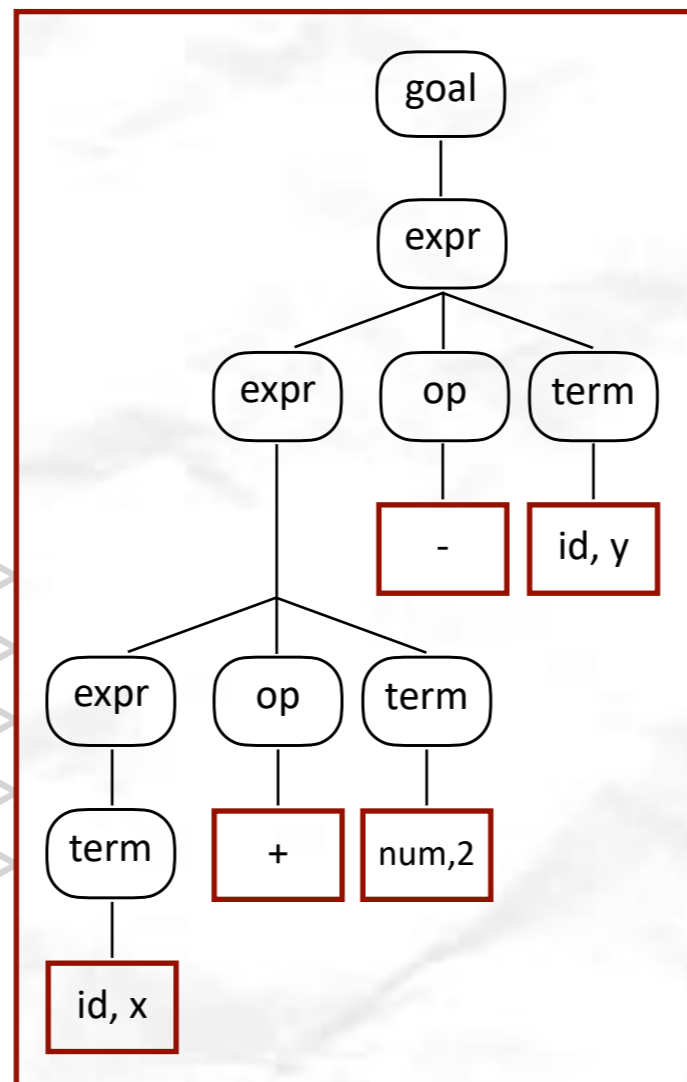
1. $\langle \text{goal} \rangle \rightarrow \langle \text{expr} \rangle$
2. $\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{term} \rangle$
3. $\langle \text{expr} \rangle \rightarrow \langle \text{term} \rangle$
4. $\langle \text{term} \rangle \rightarrow \text{num}$
5. $\langle \text{term} \rangle \rightarrow \text{id}$
6. $\langle \text{op} \rangle \rightarrow +$
7. $\langle \text{op} \rangle \rightarrow -$

input tokens: $x + 2 - y$

Left-most derivation

```

<goal>
<expr>
<expr> <op> <term>
<expr> <op> <term> <op>
<term> <op> <term> <op>
[id, x] <op> <term> <op>
[id, x] + <term> <op>
[id, x] + [num, 2] <op>
[id, x] + [num, 2] -
[id, x] + [num, 2] -
  
```



Concrete Syntax Tree

Right-most derivation

```

<goal>
<expr>
<expr> <op> <term>
<expr> <op> [id, y]
<expr> - [id, y]
<op> <term> - [id, y]
<op> [num, 2] - [id, y]
+ [num, 2] - [id, y]
+ [num, 2] - [id, y]
+ [num, 2] - [id, y]
  
```

Context-Free Grammars

Derivations

Consider this grammar from the Dragon book . . .

1. `<list> ::= <list> + <digit>`
2. `<list> ::= <list> - <digit>`
3. `<list> ::= <digit>`
4. `<digit> ::= 0|1|2|3|4|5|6|7|8|9`

. . . and input tokens: 9 - 5 + 2

Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

input tokens: 9 - 5 + 2



S. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.

list

We start with the start symbol.

Then what?

There are three $\langle \text{list} \rangle$ productions.

Which should we choose?

Let's try #3.

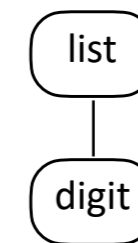
Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

- S. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.
3. $\langle \text{digit} \rangle$ Production #3 : $\langle \text{list} \rangle \rightarrow \langle \text{digit} \rangle$

input tokens: 9 - 5 + 2



Context-Free Grammars

Derivations

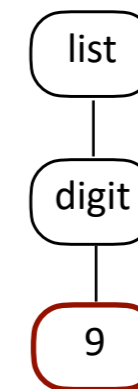
1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

5. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.
3. $\langle \text{digit} \rangle$ Production #3 : $\langle \text{list} \rangle \rightarrow \langle \text{digit} \rangle$
4. 9 Production #4 : $\langle \text{digit} \rangle \rightarrow 9$

Consume 9 from the token stream.

Move the input pointer to the next token in the stream.

input tokens: ~~X~~ - 5 + 2



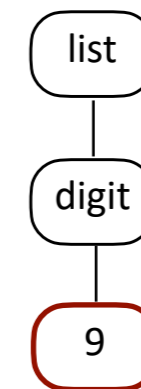
Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

5. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.
3. $\langle \text{digit} \rangle$ Production #3 : $\langle \text{list} \rangle \rightarrow \langle \text{digit} \rangle$
4. 9 Production #4 : $\langle \text{digit} \rangle \rightarrow 9$

input tokens: ~~X~~ - 5 + 2



Now what?

Context-Free Grammars

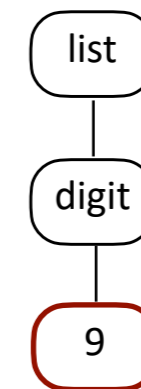
Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

input tokens: ~~X~~ - 5 + 2



5. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.
3. $\langle \text{digit} \rangle$ Production #3 : $\langle \text{list} \rangle \rightarrow \langle \text{digit} \rangle$
4. 9 Production #4 : $\langle \text{digit} \rangle \rightarrow 9$



Now what?

There are no more non-terminals to be turned into terminals, but there are still tokens on the input stream.

We must have gone down a wrong (garden?) path.

Backtrack and try a different path.

Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

input tokens: ~~9~~ - 5 + 2



5. $\langle \text{list} \rangle$
3. $\langle \text{digit} \rangle$
4. 9

Begin with the start symbol $\langle \text{list} \rangle$.

Production #3 : $\langle \text{list} \rangle \rightarrow \langle \text{digit} \rangle$

Production #4 : $\langle \text{digit} \rangle \rightarrow 9$



Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

input tokens: 9 - 5 + 2



S. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.

list

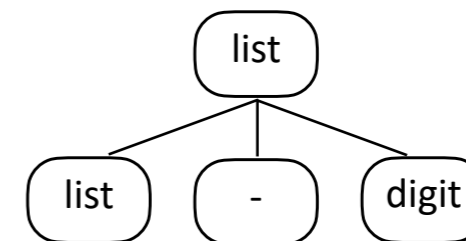
Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

5. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle$ Production #2 : $\langle \text{list} \rangle \rightarrow \langle \text{list} \rangle - \langle \text{digit} \rangle$

input tokens: 9 - 5 + 2



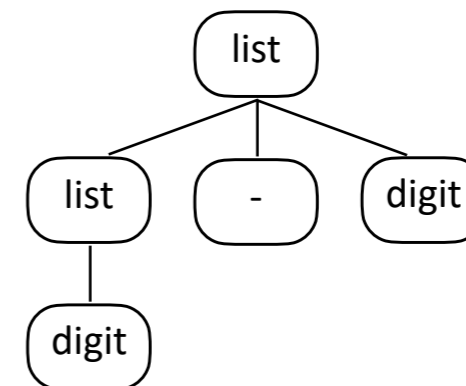
Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

5. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle$ Production #2 : $\langle \text{list} \rangle \rightarrow \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle$ Production #3 : $\langle \text{list} \rangle \rightarrow \langle \text{digit} \rangle$

input tokens: 9 - 5 + 2



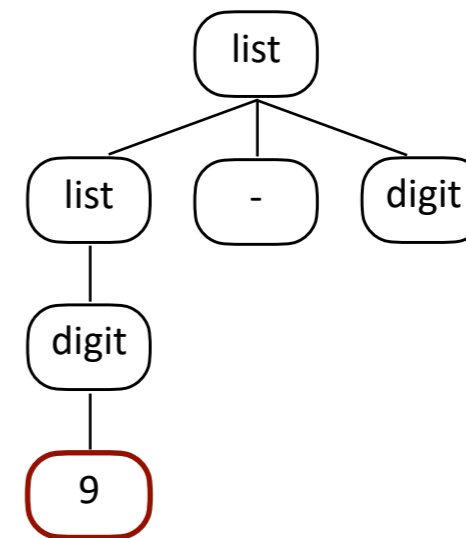
Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

5. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle$ Production #2 : $\langle \text{list} \rangle \rightarrow \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle$ Production #3 : $\langle \text{list} \rangle \rightarrow \langle \text{digit} \rangle$
4. 9 - $\langle \text{digit} \rangle$ Production #4 : $\langle \text{digit} \rangle \rightarrow 9$
 Consume 9 from the token stream.

input tokens: ~~X~~ - 5 + 2



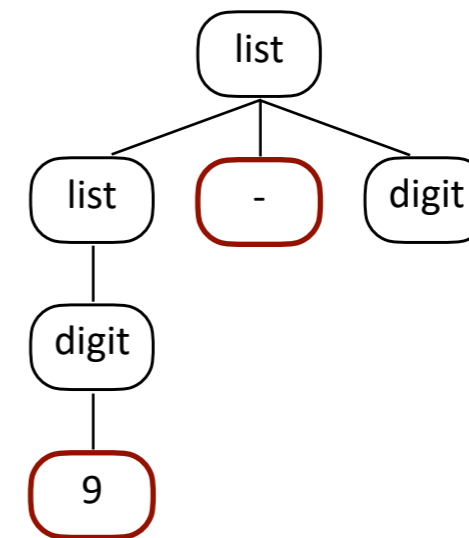
Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

5. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle$ Production #2 : $\langle \text{list} \rangle \rightarrow \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle$ Production #3 : $\langle \text{list} \rangle \rightarrow \langle \text{digit} \rangle$
4. 9 - $\langle \text{digit} \rangle$ Production #4 : $\langle \text{digit} \rangle \rightarrow 9$
 Consume 9 from the token stream.
 Consume - from the token stream.

input tokens: ~~X~~ ~~X~~ 5 + 2



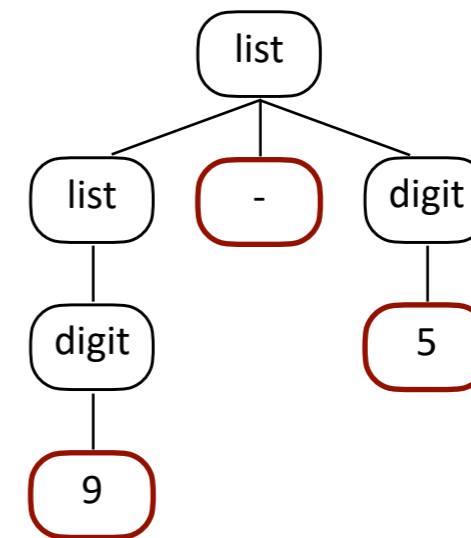
Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

5. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle$ Production #2 : $\langle \text{list} \rangle \rightarrow \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle$ Production #3 : $\langle \text{list} \rangle \rightarrow \langle \text{digit} \rangle$
4. 9 - $\langle \text{digit} \rangle$ Production #4 : $\langle \text{digit} \rangle \rightarrow 9$
4. 9 - 5 Production #4 : $\langle \text{digit} \rangle \rightarrow 5$
Consume 5 from the token stream.

input tokens: ~~X~~ ~~X~~ ~~X~~ + 2



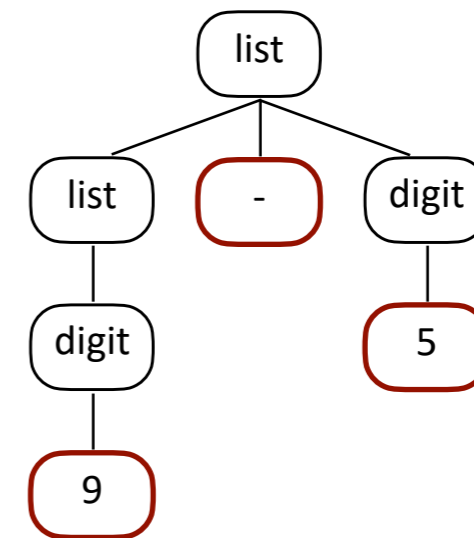
Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

5. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle$ Production #2 : $\langle \text{list} \rangle \rightarrow \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle$ Production #3 : $\langle \text{list} \rangle \rightarrow \langle \text{digit} \rangle$
4. 9 - $\langle \text{digit} \rangle$ Production #4 : $\langle \text{digit} \rangle \rightarrow 9$
4. 9 - 5 Production #4 : $\langle \text{digit} \rangle \rightarrow 5$

input tokens: ~~X~~ ~~X~~ ~~X~~ + 2



Now what?

Oops. We must have gone down another wrong path.
Backtrack and try again.

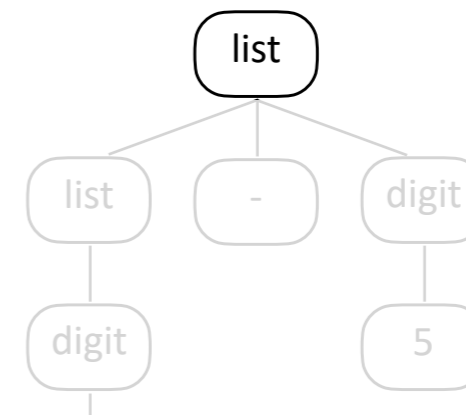
Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

input tokens: ~~9~~ ~~-~~ ~~5~~ + 2

5. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle$ Production #2 : $\langle \text{list} \rangle \rightarrow \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle$ Production #3 : $\langle \text{list} \rangle \rightarrow \langle \text{digit} \rangle$
4. 9 - $\langle \text{digit} \rangle$ Production #4 : $\langle \text{digit} \rangle \rightarrow 9$
4. 9 - 5 Production #4 : $\langle \text{digit} \rangle \rightarrow 5$



Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

input tokens: 9 - 5 + 2



S. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.

list

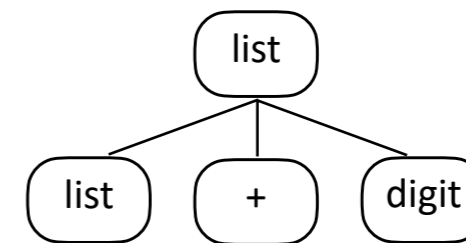
Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

- S. $\langle \text{list} \rangle$ Begin with the start symbol $\langle \text{list} \rangle$.
1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$ Production #1 : $\langle \text{list} \rangle \rightarrow \langle \text{list} \rangle + \langle \text{digit} \rangle$

input tokens: 9 - 5 + 2



Context-Free Grammars

Derivations

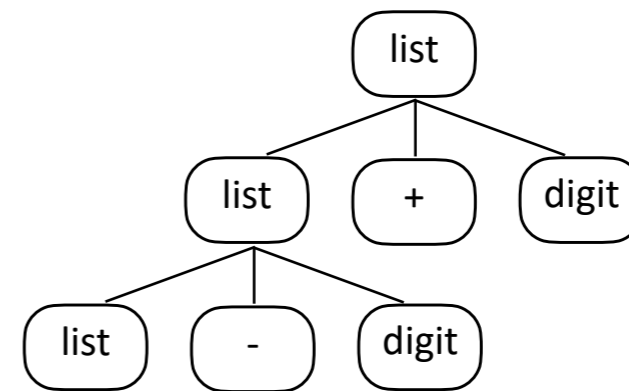
1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0|1|2|3|4|5|6|7|8|9$

S. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$

2. $\langle \text{list} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$

input tokens: 9 - 5 + 2



Context-Free Grammars

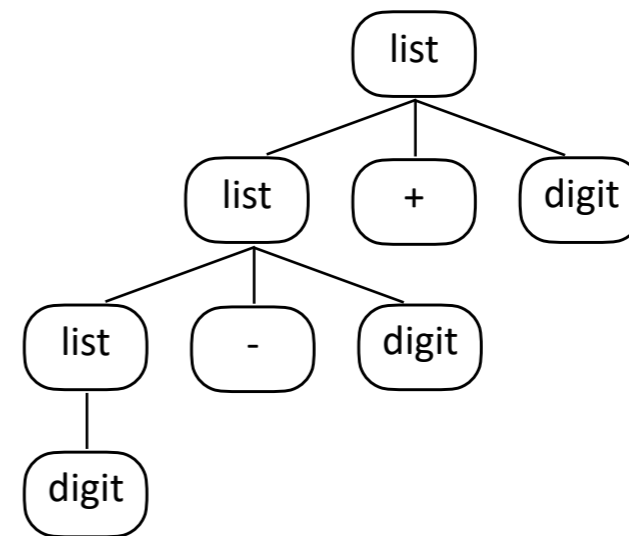
Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

S. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$

input tokens: 9 - 5 + 2



Context-Free Grammars

Derivations

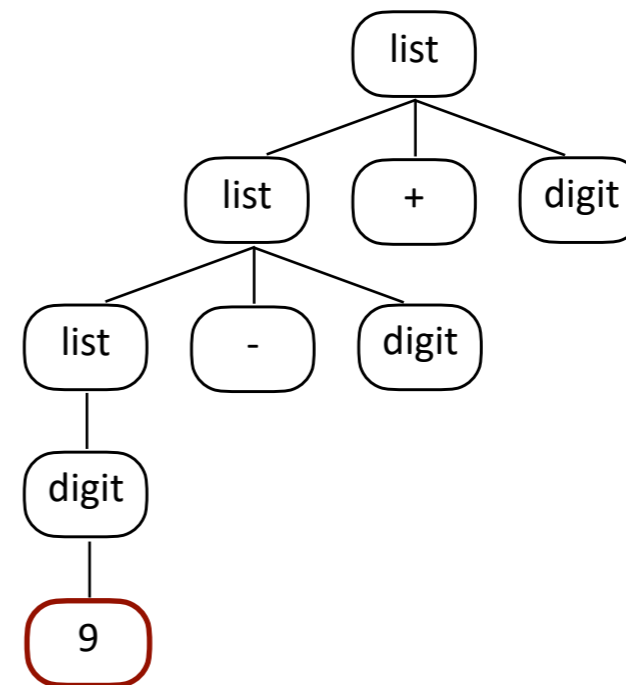
1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0|1|2|3|4|5|6|7|8|9$

5. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
4. $\boxed{9} - \langle \text{digit} \rangle + \langle \text{digit} \rangle$

Consume 9 from the token stream.

input tokens: ~~X~~ - 5 + 2



Context-Free Grammars

Derivations

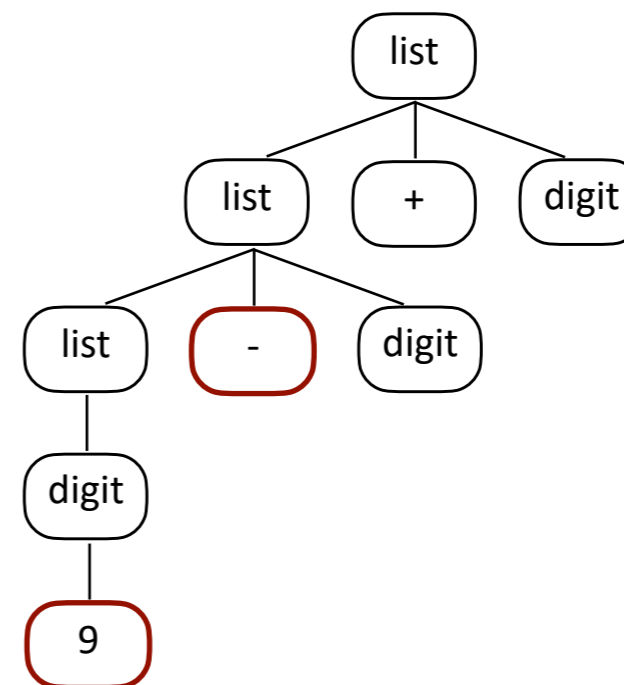
1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

5. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
4. $\boxed{9} \boxed{-} \langle \text{digit} \rangle + \langle \text{digit} \rangle$

Consume - from the token stream.

input tokens: ~~X~~ ~~X~~ 5 + 2



Context-Free Grammars

Derivations

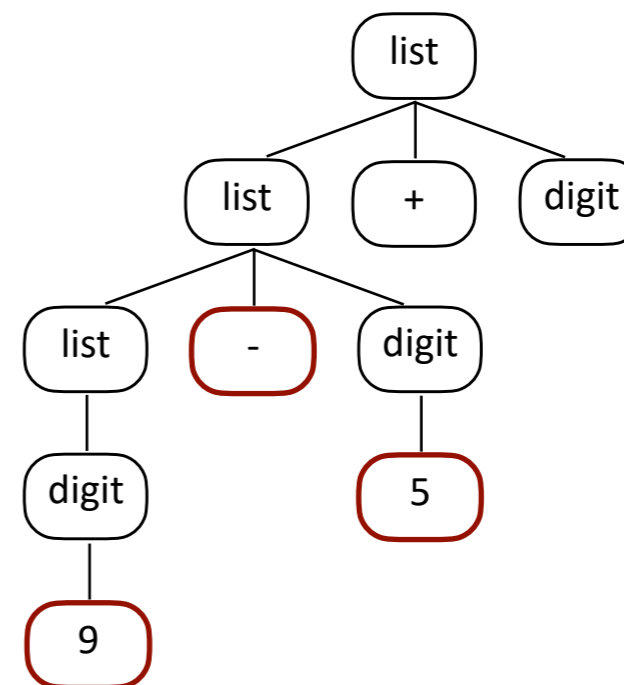
1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0|1|2|3|4|5|6|7|8|9$

5. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
4. $\boxed{9} \quad \boxed{-} \quad \langle \text{digit} \rangle + \langle \text{digit} \rangle$
4. $\boxed{9} \quad \boxed{-} \quad \boxed{5} \quad + \langle \text{digit} \rangle$

Consume 5 from the token stream.

input tokens: ~~X~~ ~~X~~ ~~X~~ + 2



Context-Free Grammars

Derivations

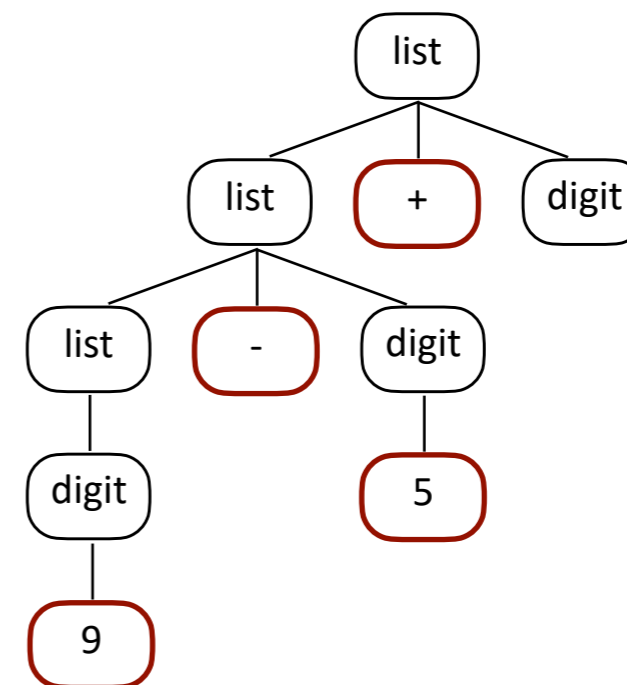
1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

5. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
4. $\boxed{9} \boxed{-} \langle \text{digit} \rangle + \langle \text{digit} \rangle$
4. $\boxed{9} \boxed{-} \boxed{5} \boxed{+} \langle \text{digit} \rangle$

Consume + from the token stream.

input tokens: ~~X~~ ~~X~~ ~~X~~ ~~X~~ 2



Context-Free Grammars

Derivations

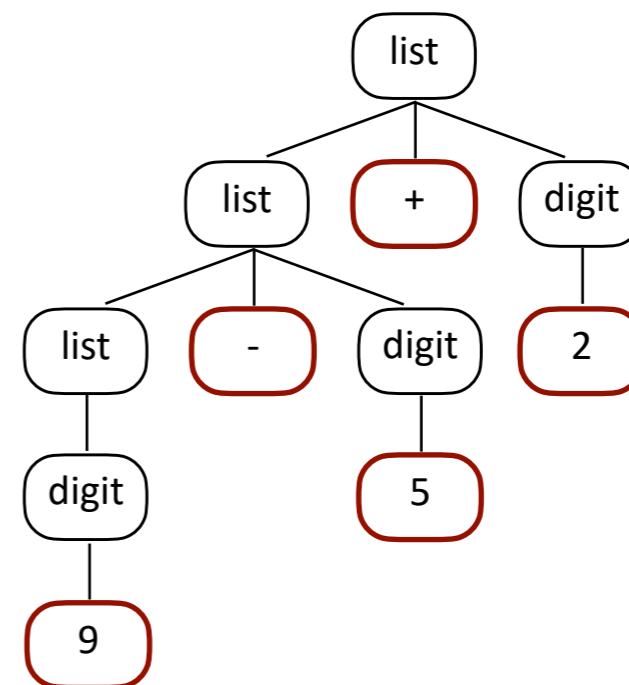
1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

5. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
4. $\boxed{9} \quad \boxed{-} \quad \langle \text{digit} \rangle + \langle \text{digit} \rangle$
4. $\boxed{9} \quad \boxed{-} \quad \boxed{5} \quad \boxed{+} \quad \langle \text{digit} \rangle$
4. $\boxed{9} \quad \boxed{-} \quad \boxed{5} \quad \boxed{+} \quad \boxed{2}$

Consume 2 from the token stream.

input tokens: ~~X~~ ~~X~~ ~~X~~ ~~X~~ ~~X~~



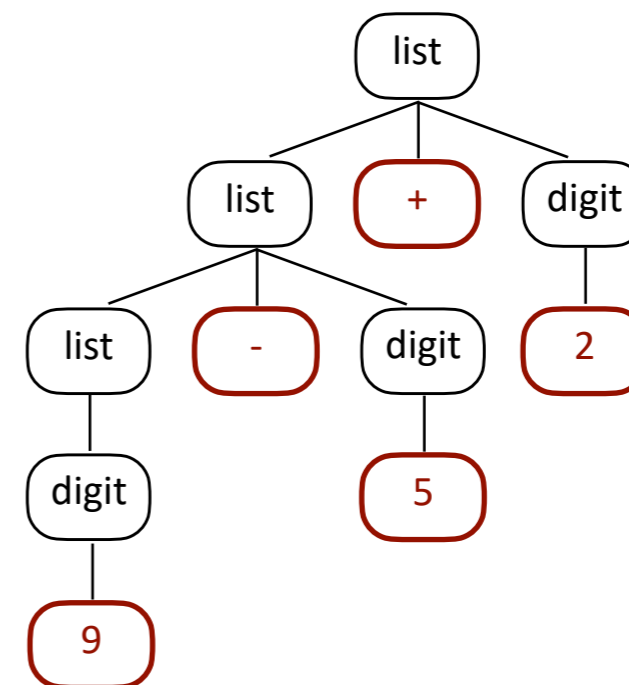
Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

input tokens: ~~X~~ ~~X~~ ~~X~~ ~~X~~ ~~X~~

5. $\langle \text{list} \rangle$
1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$
 2. $\langle \text{list} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
 3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
 4. $\boxed{9} \quad \boxed{-} \quad \langle \text{digit} \rangle + \langle \text{digit} \rangle$
 4. $\boxed{9} \quad \boxed{-} \quad \boxed{5} \quad \boxed{+} \quad \langle \text{digit} \rangle$
 4. $\boxed{9} \quad \boxed{-} \quad \boxed{5} \quad \boxed{+} \quad \boxed{2}$



Now what?

We're good! We've run out of non-terminals to turn into terminals at the same time as we've run out of input tokens to process. This is a beautiful thing... a successful parse.

Context-Free Grammars

Left-most Derivation

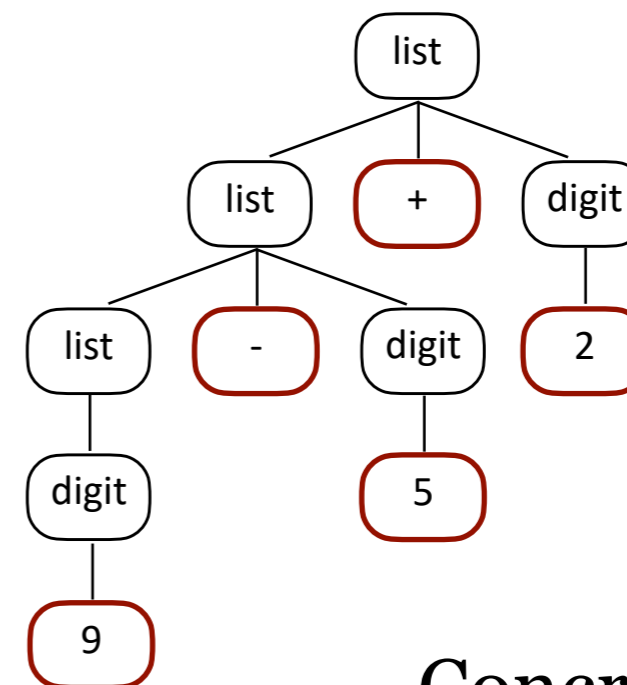
1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0|1|2|3|4|5|6|7|8|9$

5. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
4. $9 - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
4. $9 - 5 + \langle \text{digit} \rangle$
4. $9 - 5 + 2$

This is a **left-most** derivation.
Meaning... we processed non-terminals from left to right.

input tokens: ~~X~~ ~~X~~ ~~X~~ ~~X~~ ~~X~~



Concrete
Syntax Tree
(CST)

Context-Free Grammars

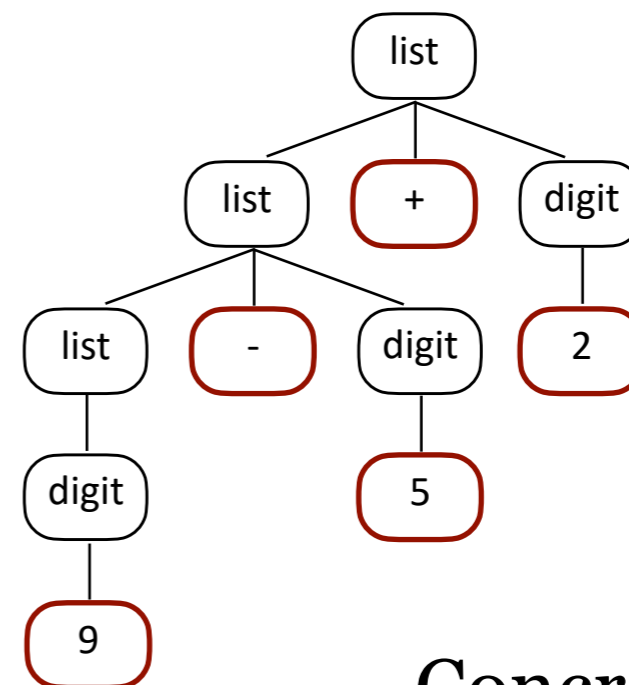
Left-most Derivation

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

5. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
4. $9 - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
4. $9 - 5 + \langle \text{digit} \rangle$
4. $9 - 5 + 2$

input tokens: ~~X~~ ~~X~~ ~~X~~ ~~X~~ ~~X~~



Concrete
Syntax Tree
(CST)

This is a **left-most** derivation.

Let's try a right-most derivation.

Context-Free Grammars

Right-most Derivation

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

S. $\langle \text{list} \rangle$

input tokens: 9 - 5 + 2



list

Context-Free Grammars

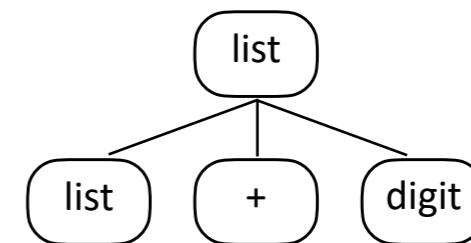
Right-most Derivation

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

S. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle$ $+ \langle \text{digit} \rangle$

input tokens: 9 - 5 + 2



Context-Free Grammars

Right-most Derivation

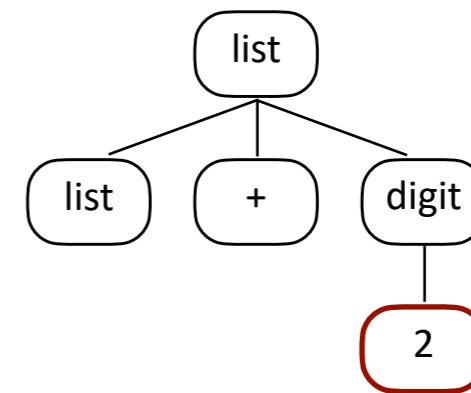
1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

S. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$

4. $\langle \text{list} \rangle + \boxed{2}$

input tokens: 9 - 5 + ~~X~~



Context-Free Grammars

Right-most Derivation

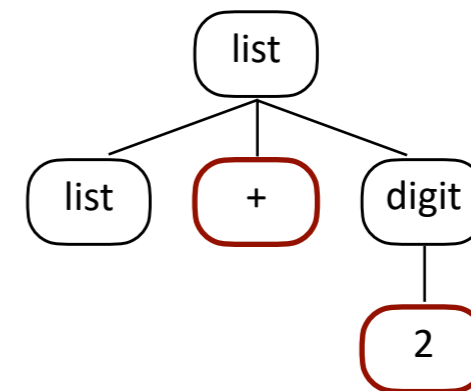
1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

S. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle$ $+ \langle \text{digit} \rangle$

4. $\langle \text{list} \rangle$ $\boxed{+}$ $\boxed{2}$

input tokens: 9 - 5 ~~X~~ ~~X~~



Context-Free Grammars

Right-most Derivation

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

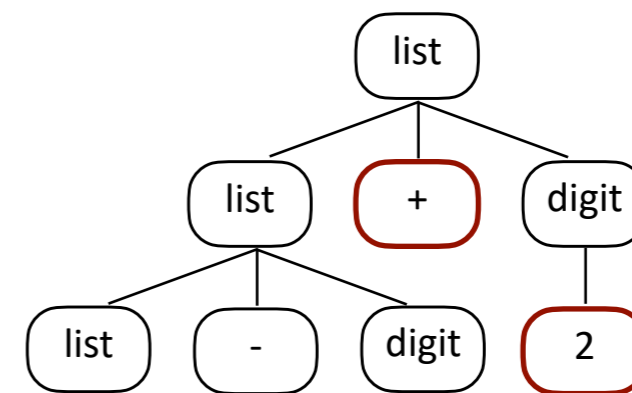
S. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle$ + $\langle \text{digit} \rangle$

4. $\langle \text{list} \rangle$ + 2

2. $\langle \text{list} \rangle$ - $\langle \text{digit} \rangle$ + 2

input tokens: 9 - 5 ~~X~~ ~~X~~



Context-Free Grammars

Right-most Derivation

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

S. $\langle \text{list} \rangle$

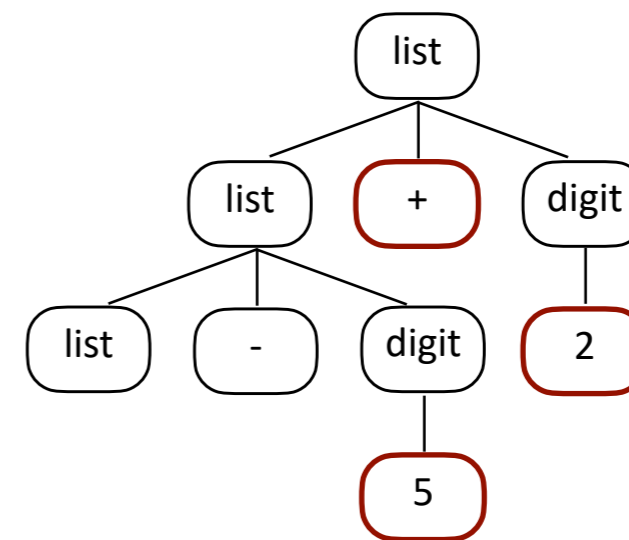
1. $\langle \text{list} \rangle$ + $\langle \text{digit} \rangle$

4. $\langle \text{list} \rangle$ + $\boxed{+}$ $\boxed{2}$

2. $\langle \text{list} \rangle$ - $\langle \text{digit} \rangle$ + $\boxed{+}$ $\boxed{2}$

4. $\langle \text{list} \rangle$ - $\boxed{5}$ + $\boxed{+}$ $\boxed{2}$

input tokens: 9 - ~~X~~ ~~X~~ ~~X~~



Context-Free Grammars

Right-most Derivation

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0|1|2|3|4|5|6|7|8|9$

S. $\langle \text{list} \rangle$

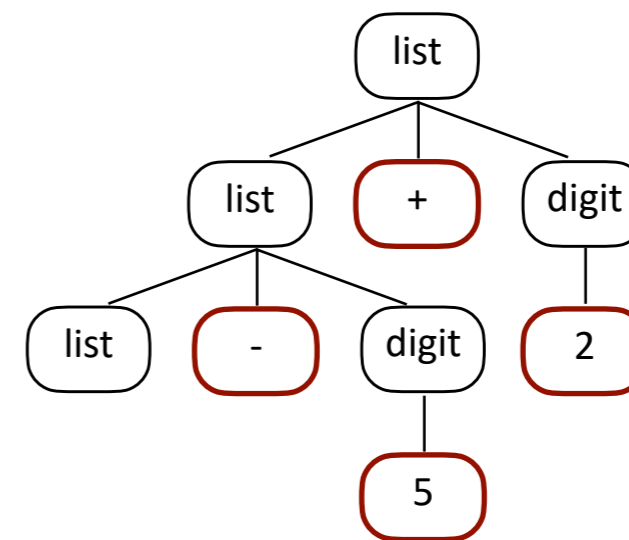
1. $\langle \text{list} \rangle$ + $\langle \text{digit} \rangle$

4. $\langle \text{list} \rangle$ $\boxed{+}$ $\boxed{2}$

2. $\langle \text{list} \rangle$ - $\langle \text{digit} \rangle$ $\boxed{+}$ $\boxed{2}$

4. $\langle \text{list} \rangle$ $\boxed{-}$ $\boxed{5}$ $\boxed{+}$ $\boxed{2}$

input tokens: 9 ~~X~~ ~~X~~ ~~X~~ ~~X~~



Context-Free Grammars

Right-most Derivation

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0|1|2|3|4|5|6|7|8|9$

5. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle$ + $\langle \text{digit} \rangle$

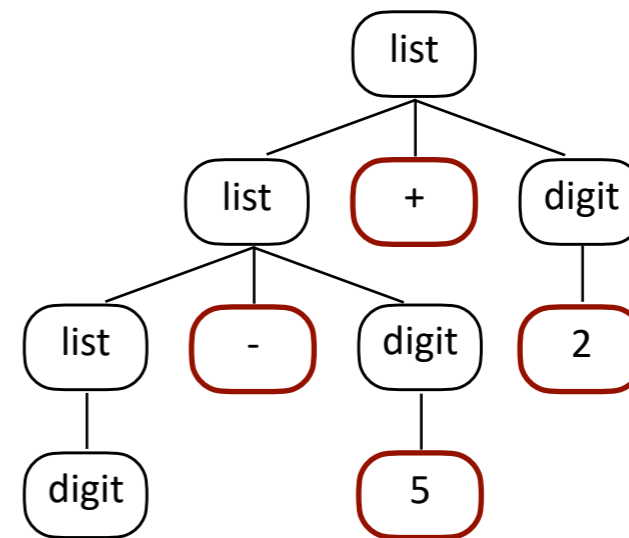
4. $\langle \text{list} \rangle$ $\boxed{+}$ $\boxed{2}$

2. $\langle \text{list} \rangle$ - $\langle \text{digit} \rangle$ $\boxed{+}$ $\boxed{2}$

4. $\langle \text{list} \rangle$ $\boxed{-}$ $\boxed{5}$ $\boxed{+}$ $\boxed{2}$

3. $\langle \text{digit} \rangle$ $\boxed{-}$ $\boxed{5}$ $\boxed{+}$ $\boxed{2}$

input tokens: 9 ~~X~~ ~~X~~ ~~X~~ ~~X~~



Context-Free Grammars

Right-most Derivation

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

5. $\langle \text{list} \rangle$

1. $\langle \text{list} \rangle$ + $\langle \text{digit} \rangle$

4. $\langle \text{list} \rangle$ + $\boxed{2}$

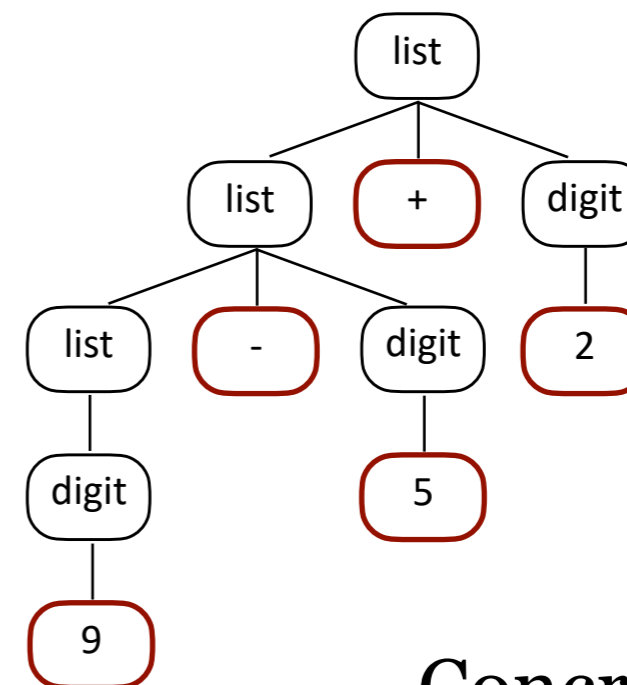
2. $\langle \text{list} \rangle$ - $\langle \text{digit} \rangle$ + $\boxed{2}$

4. $\langle \text{list} \rangle$ - $\boxed{5}$ + $\boxed{2}$

3. $\langle \text{digit} \rangle$ - $\boxed{5}$ + $\boxed{2}$

4. $\boxed{9}$ - $\boxed{5}$ + $\boxed{2}$

input tokens: ~~X~~ ~~X~~ ~~X~~ ~~X~~ ~~X~~



Concrete
Syntax Tree
(CST)

Context-Free Grammars

Right-most Derivation

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$

input tokens: ~~X~~ ~~X~~ ~~X~~ ~~X~~ ~~X~~

5. $\langle \text{list} \rangle$
1. $\langle \text{list} \rangle$ + $\langle \text{digit} \rangle$
 4. $\langle \text{list} \rangle$

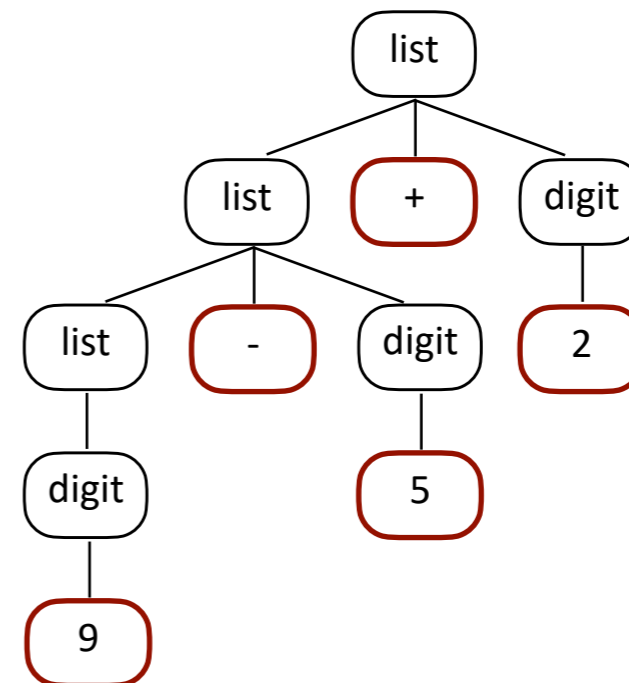
+	2
---	---
 2. $\langle \text{list} \rangle$ - $\langle \text{digit} \rangle$

+	2
---	---
 4. $\langle \text{list} \rangle$

-	5	+	2
---	---	---	---
 3. $\langle \text{digit} \rangle$

-	5	+	2
---	---	---	---
 4.

9	-	5	+	2
---	---	---	---	---



We've run out of non-terminals to turn into terminals at the same time as we've run out of input tokens to process. This is a beautiful thing... another successful parse, this time a right-most derivation.

Context-Free Grammars

Derivations

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0|1|2|3|4|5|6|7|8|9$

input tokens: ~~X~~ ~~X~~ ~~X~~ ~~X~~ ~~X~~

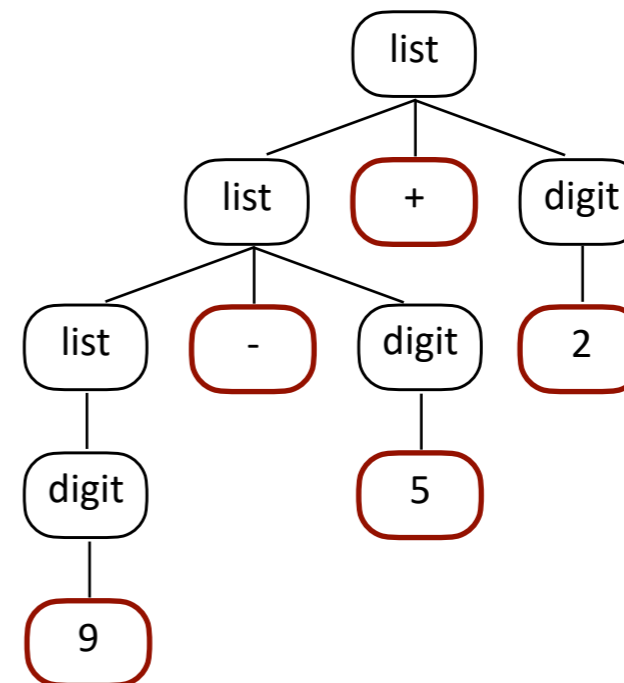
Earlier slide

1. $\langle \text{list} \rangle ::= \langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle ::= \langle \text{list} \rangle - \langle \text{digit} \rangle$
3. $\langle \text{list} \rangle ::= \langle \text{digit} \rangle$
4. $\langle \text{digit} \rangle ::= 0|1|2|3|4|5|6|7|8|9$

input tokens: ~~X~~ ~~X~~ ~~X~~ ~~X~~ ~~X~~

5. $\langle \text{list} \rangle$
1. $\langle \text{list} \rangle + \langle \text{digit} \rangle$
2. $\langle \text{list} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
3. $\langle \text{digit} \rangle - \langle \text{digit} \rangle + \langle \text{digit} \rangle$
4. $\boxed{9} - \boxed{5} + \langle \text{digit} \rangle$
4. $\boxed{9} - \boxed{5} + \boxed{2}$
4. $\boxed{9} - \boxed{5} + \boxed{2}$

Now what?
We're good! We've run out of non-terminals to turn into terminals at the same time as we've run out of input tokens to process. This is a beautiful thing... a successful parse.



Note the CST resulting from the left-most derivation is the same as the CST resulting from the right-most derivation. It's nice when that works out, as the grammar is unambiguous.