Theory of Programming Languages

CMPT 331

Fun with Mbnceb Dbmdvmvt - 100 points

Goal

To appreciate the joy of the Lambda Calculus as a fundamental model of computation.

Problems

Each part of each problem is worth 10 points [1].

1. Beta-reduce the following expressions to their normal form:
   a. (λa λy . y a) (z z)
   b. (λx λy.(x y)) (λz.y)
   c. (λx.(x x)) (λy.(y y))
   d. K x y (Expand K in this problem and then you don't have to in later ones.)
   e. S K
   f. (S K) y y z
   g. K' y y z

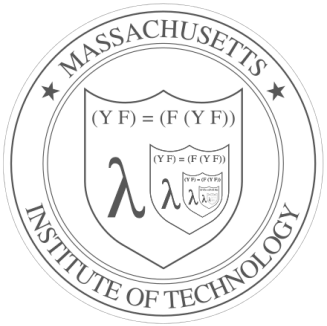
2. What is the normal form of (K S) (K I)?

3. Prove the following equivalencies by reducing each side to its normal form.
   a. I = S K K
   b. S K K = K I I

4. Given the definition of Church numerals below, what does (m n) do when m and n are Church numerals? For example ($\overline{2}$ $\overline{3}$). It may be easier to work out as λm λn . (m n) . Show your work (or at least an example).

Church Numerals

Let $\overline{0}$ = λf x . x
Let $\overline{1}$ = λf x . (f x)
Let $\overline{2}$ = λf x . (f (f x))
Let $\overline{3}$ = λf x . (f (f (f x)))
.
.
.
.
.
.
.
.
.



Let $\overline{n}$ = λf x . (f^n. x)

Let successor = succ = λn f x . n f (f x)

(succ $\overline{0}$) ≡ (λn f x . n f (f x)) ($\overline{0}$)
[$\overline{0}$/n] in λ f x . n f (f x)
→β λ f x . $\overline{0}$ f (f x)

The result is going to be λ f x . *something*.

something is $\overline{0}$ f (f x)
≡ (λf x . x) f (f x)
[f/f in λ x . x]
→β λ x . x (f x)
[(f x) / x in x]
→β (f x)

plugging *something* back into the above…
λf x . (f x)
≡ $\overline{1}$

Submitting

Write your answers using LaTeX. (See template and resources on our web site.) **Compile** your document into a PDF and print it out to hand in on or before the class in which it is due. Remember to include your name. Also, **you must show all work**. This is college. If you find my request to show your work surprising, drop this class immediately.

[1] Yes, I know it adds up to 110 points. You can make a mistake or two and still get a good score. Who loves ya?