

MIT 6.035
Introduction to Shift-Reduce Parsing

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Orientation

- Specify Syntax Using Context-Free Grammar
 - Nonterminals
 - Terminals
 - Productions
- Given a grammar, Parser Generator produces a parser
 - Starts with input string
 - Produces parse tree

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Pushdown Automata

- Consists of
 - Pushdown stack (can have terminals and nonterminals)
 - Finite state automaton control
- Can do one of three actions (based on state and input):
 - Shift:
 - Shift current input symbol from input onto stack
 - Reduce:
 - If symbols on top of stack match right hand side of some grammar production $NT \rightarrow \beta$
 - Pop symbols (β) off of the stack
 - Push left hand side nonterminal (NT) onto stack
 - Accept the input string

Shift-Reduce Parser Example

Stack

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Input String

num	*	(num	+	num)
-----	---	---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

num	*	(num	+	num)
-----	---	---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

SHIFT

num	*	(num	+	num)
-----	---	---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

num

SHIFT

*	(num	+	num)
---	---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

num

REDUCE

*	(num	+	num)
---	---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Expr

REDUCE

num

*	(num	+	num)
---	---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

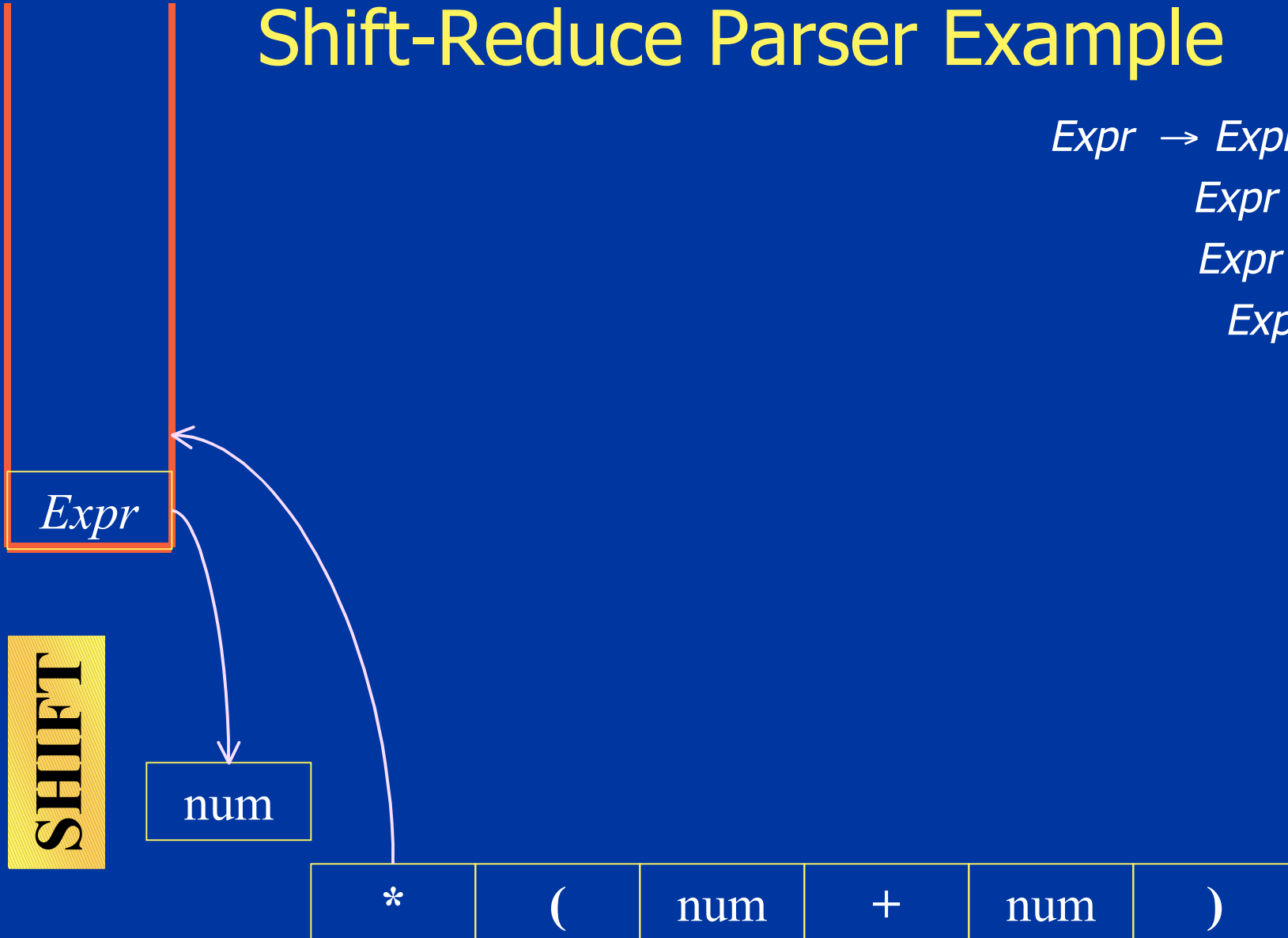
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

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$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

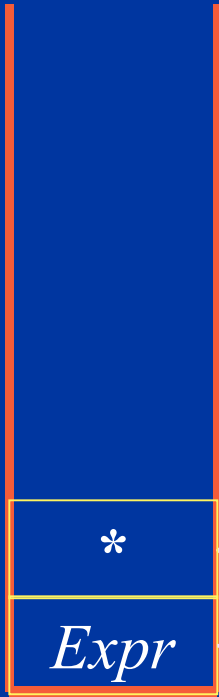
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



SHIFT

num

(num	+	num)
---	-----	---	-----	---

Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

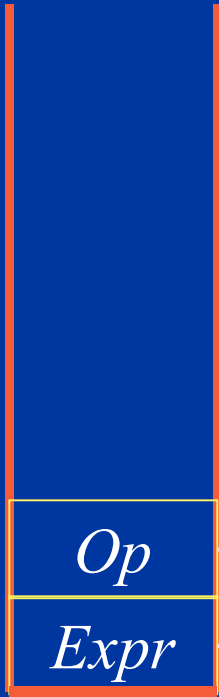
$Expr \rightarrow - Expr$

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$Op \rightarrow *$



REDUCE



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

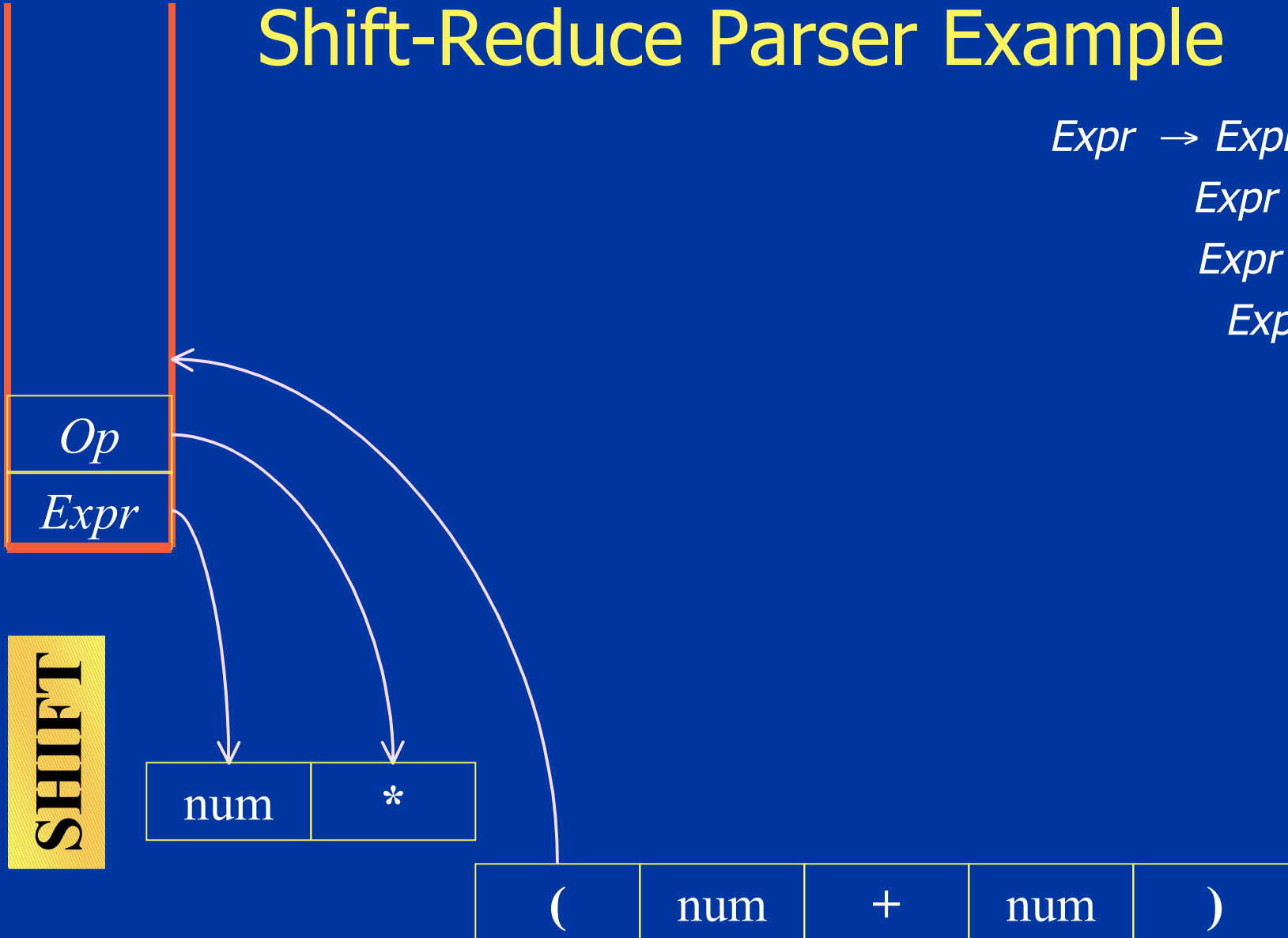
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Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

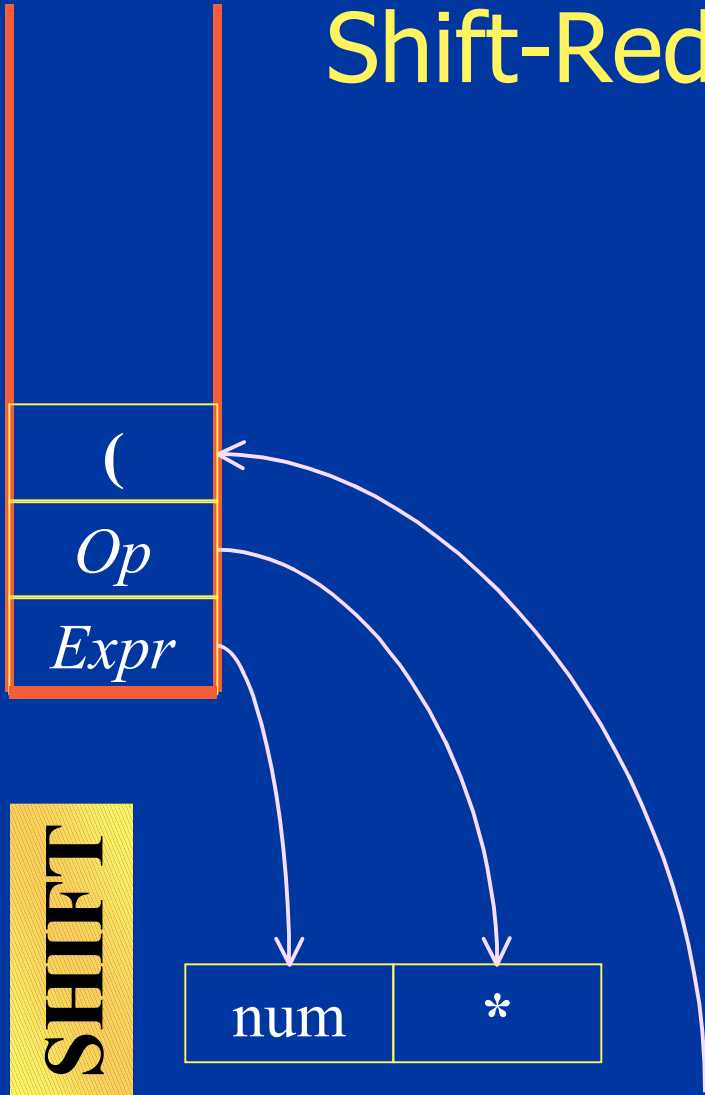
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Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

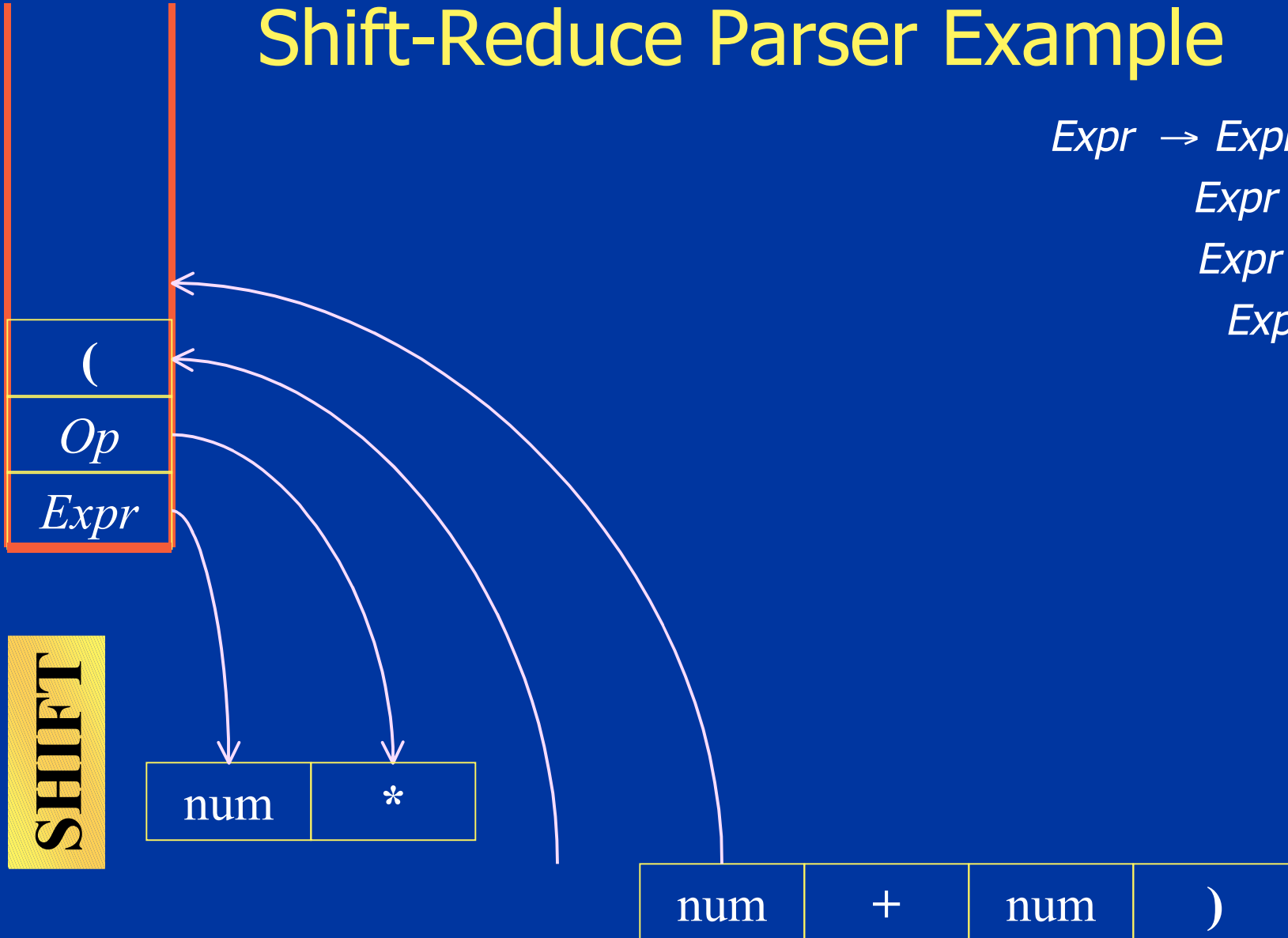
$Expr \rightarrow - Expr$

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Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

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$Expr \rightarrow - Expr$

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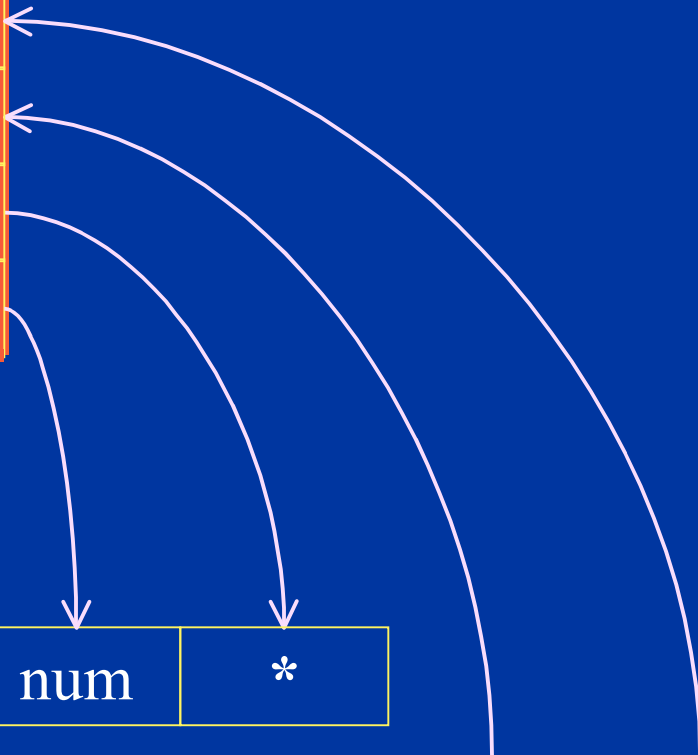
$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



SHIFT



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

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REDUCE



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

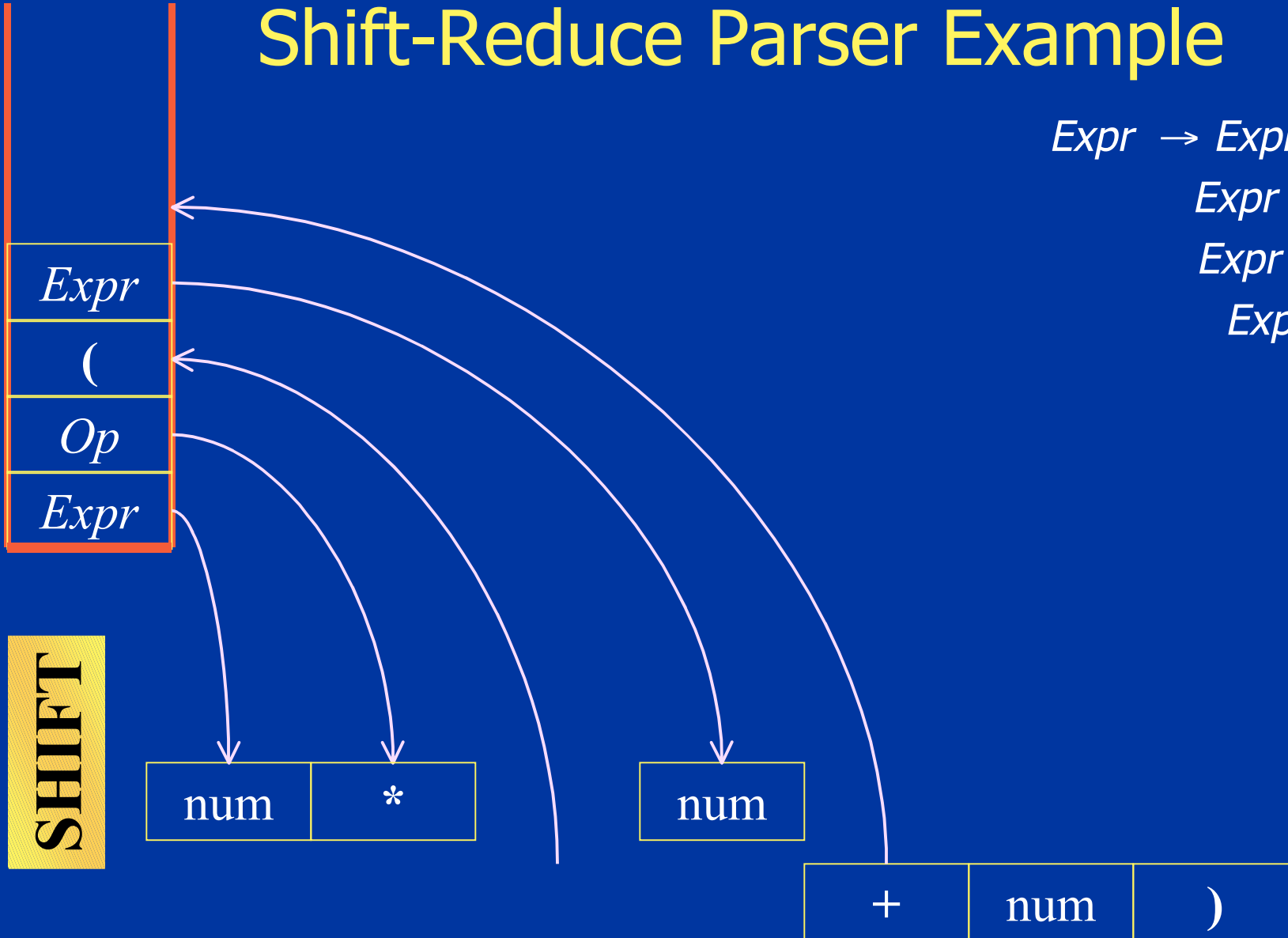
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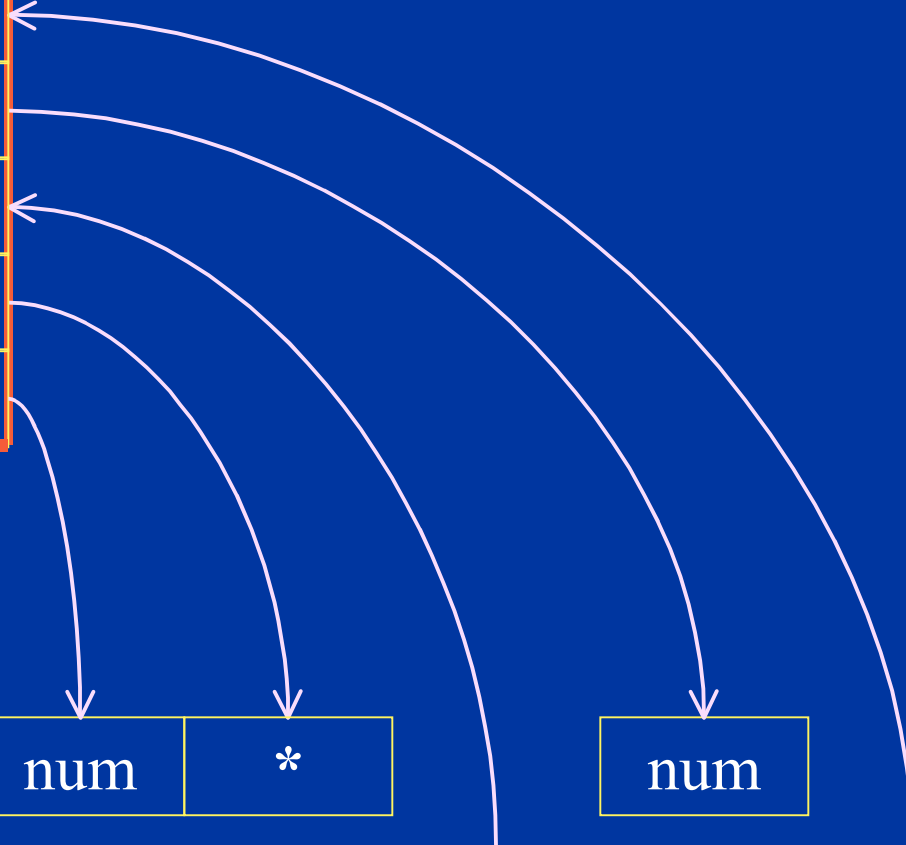
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$Op \rightarrow -$

$Op \rightarrow *$



SHIFT



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REDUCE



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$Op \rightarrow +$

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$Op \rightarrow *$



SHIFT



Shift-Reduce Parser Example



$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

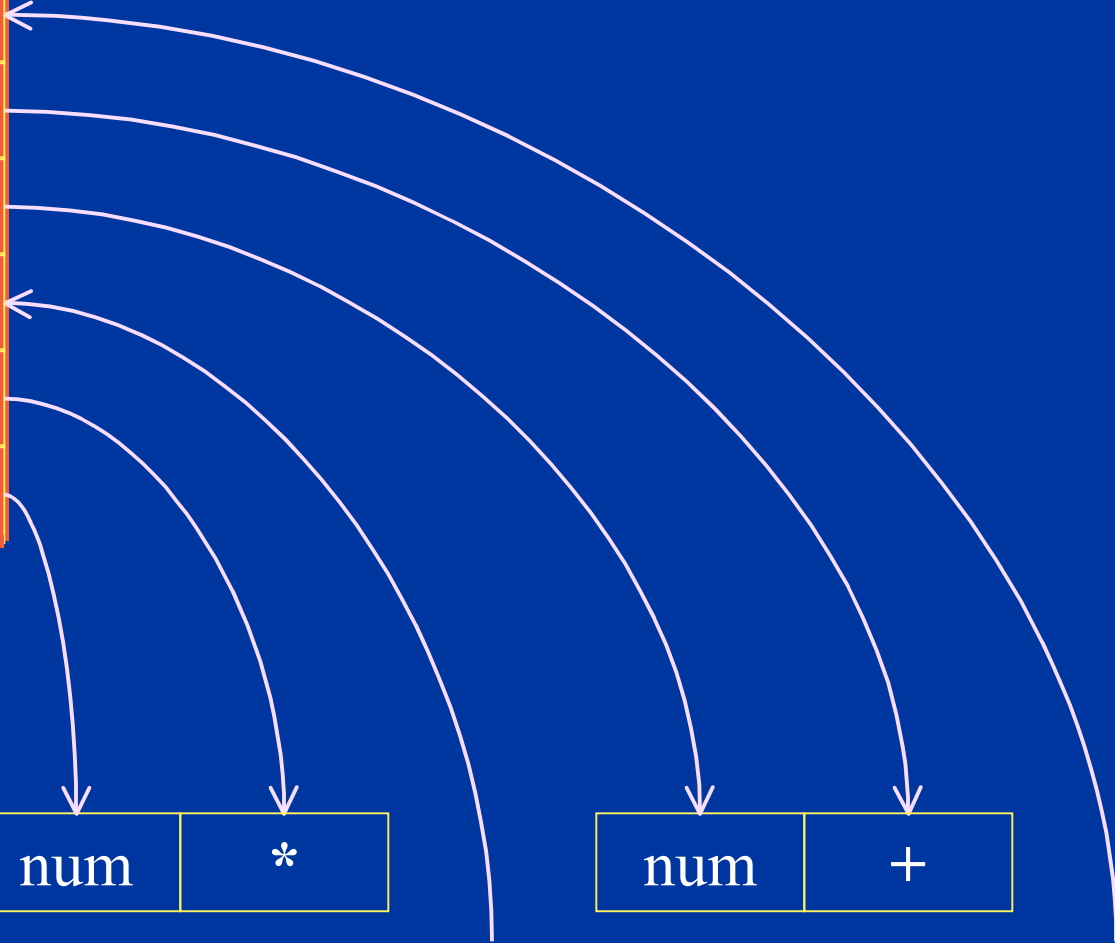
$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

SHIFT



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

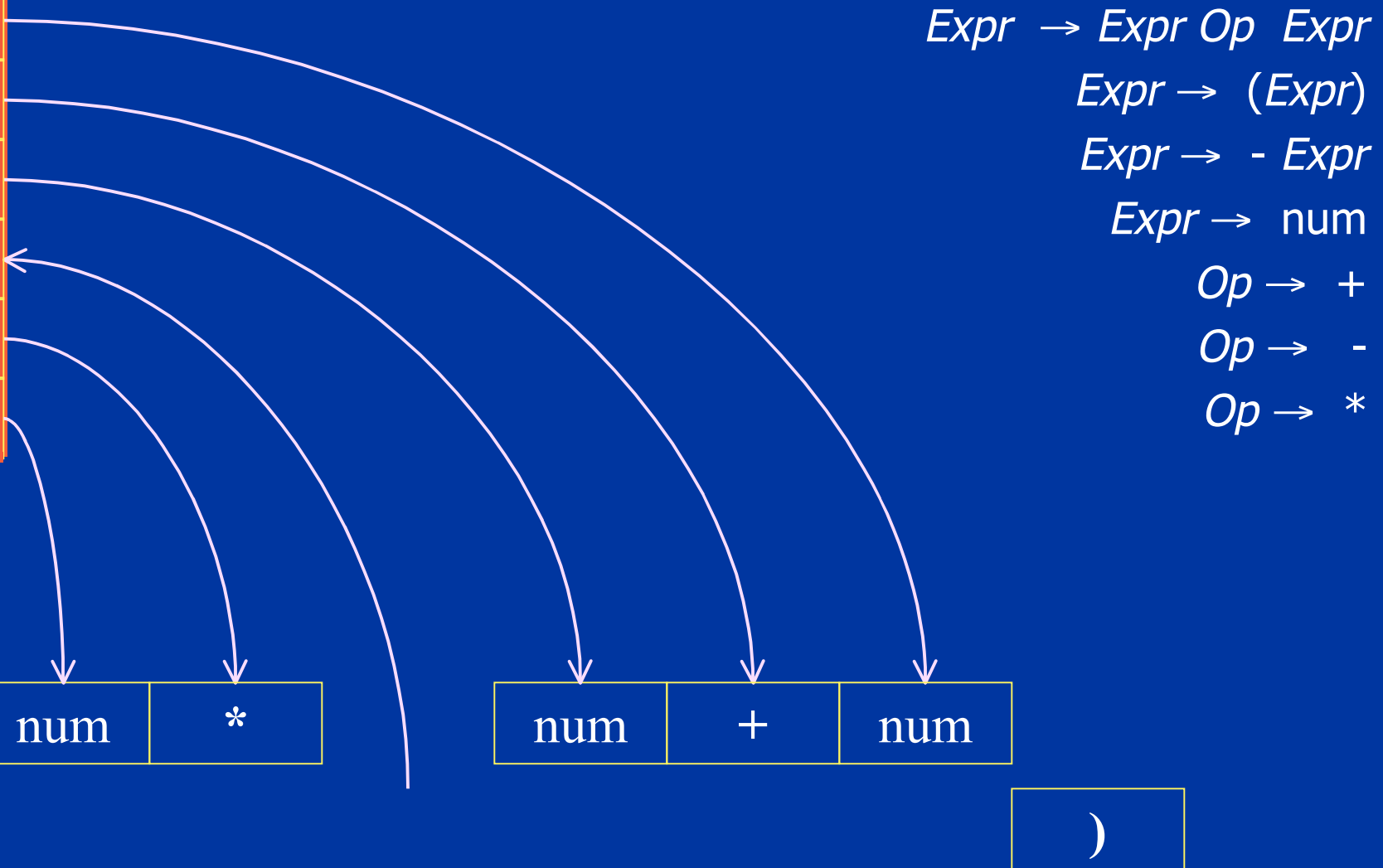
$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



REDUCE



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

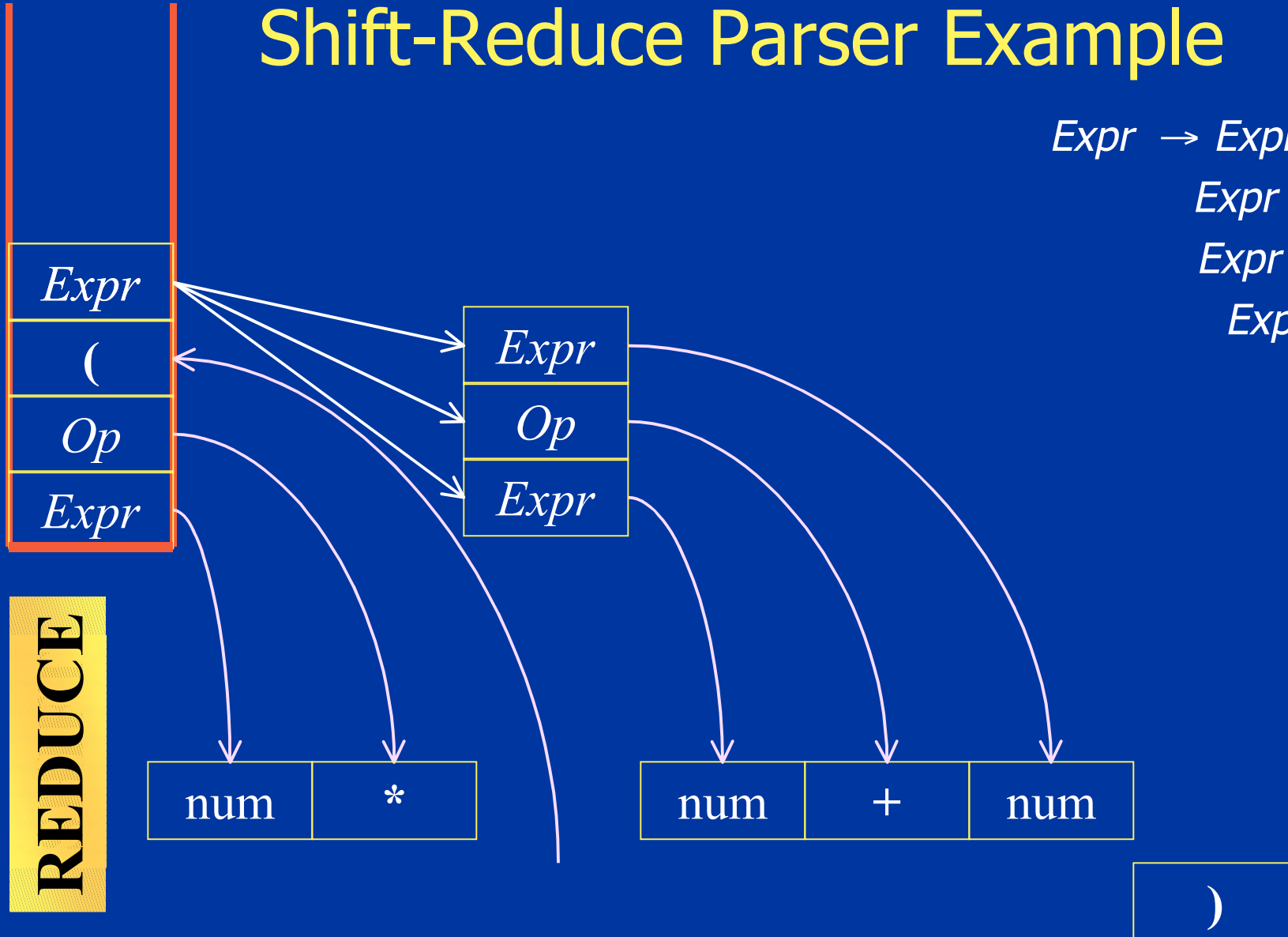
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

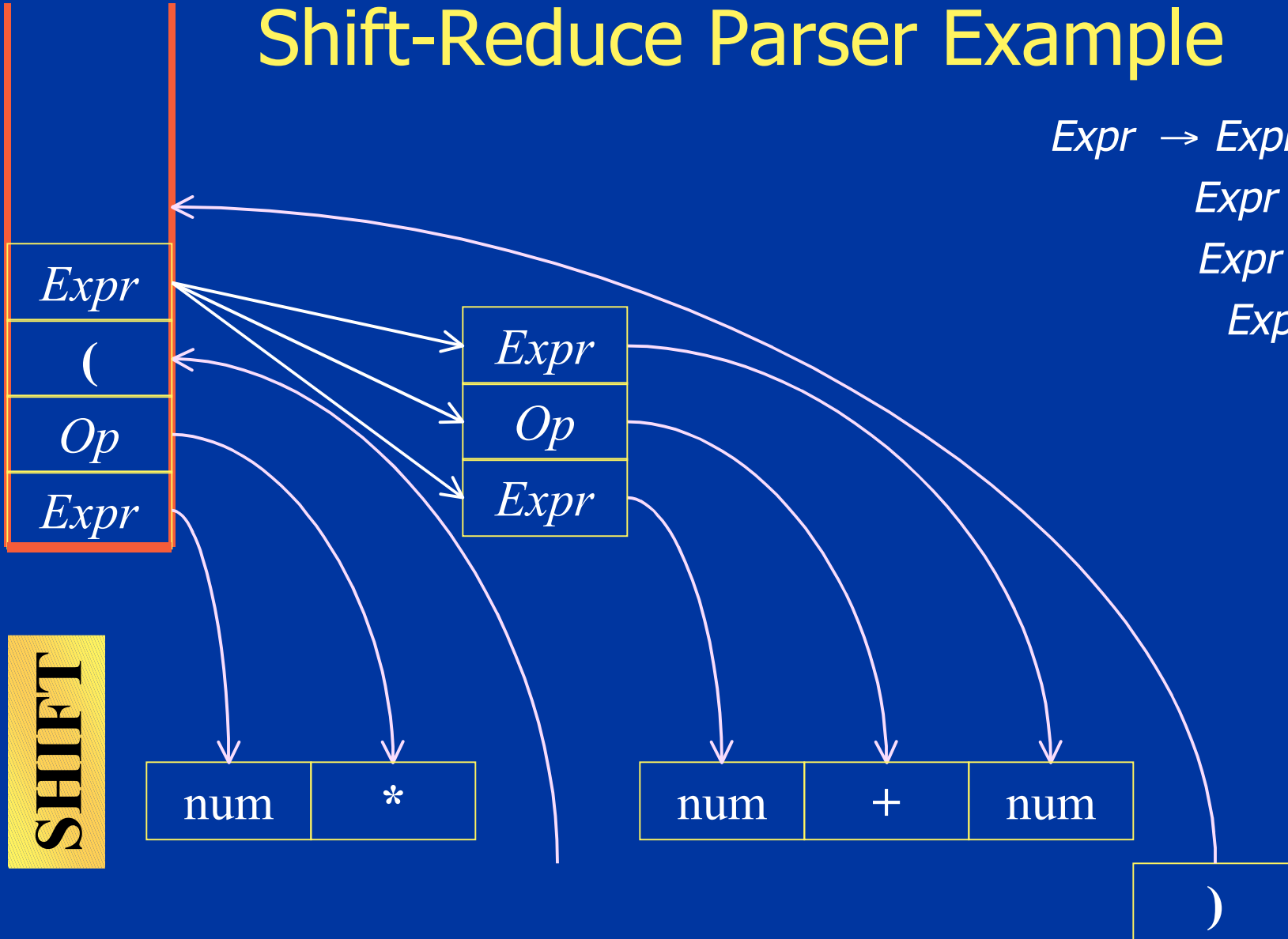
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

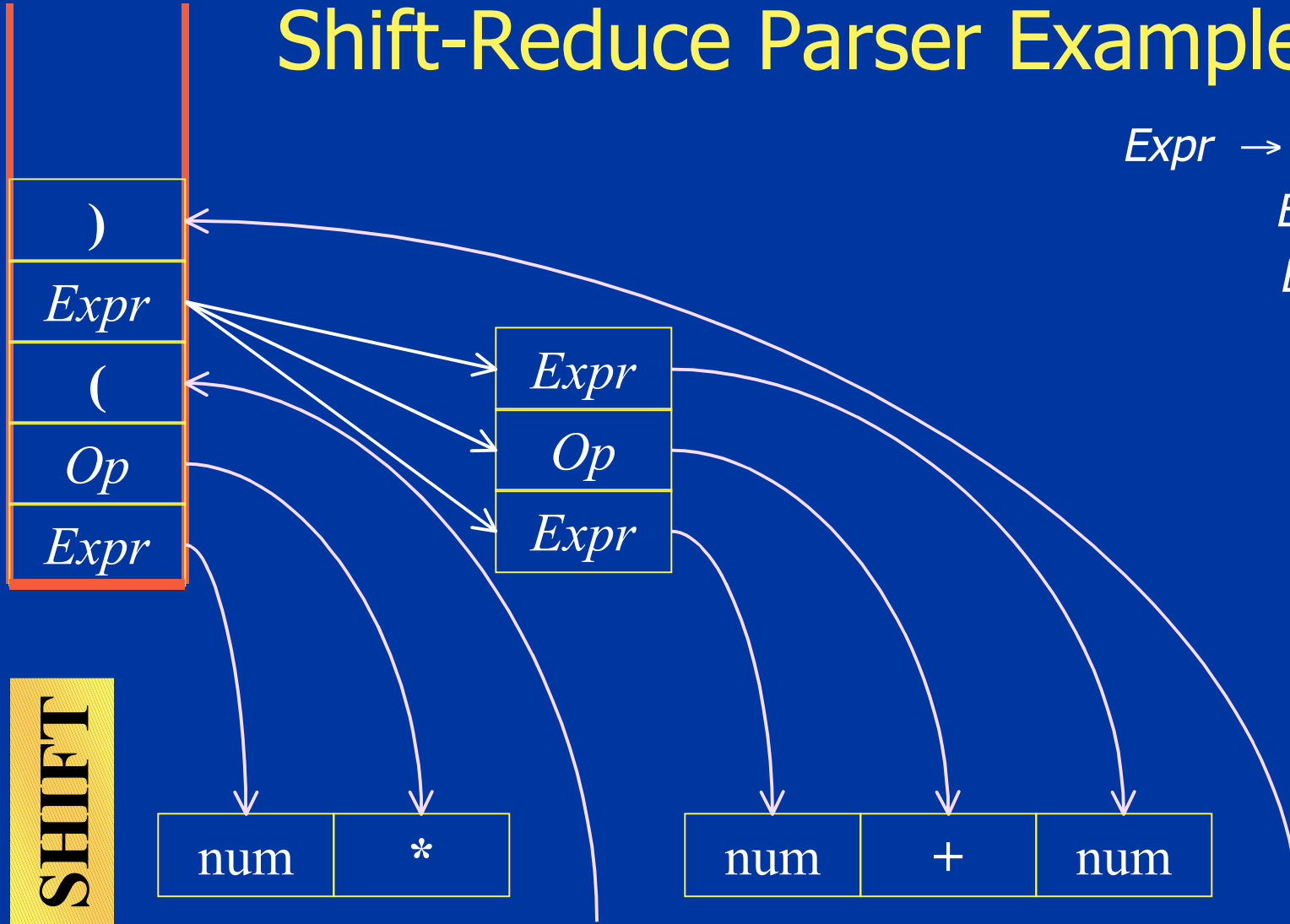
$Expr \rightarrow - Expr$

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Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

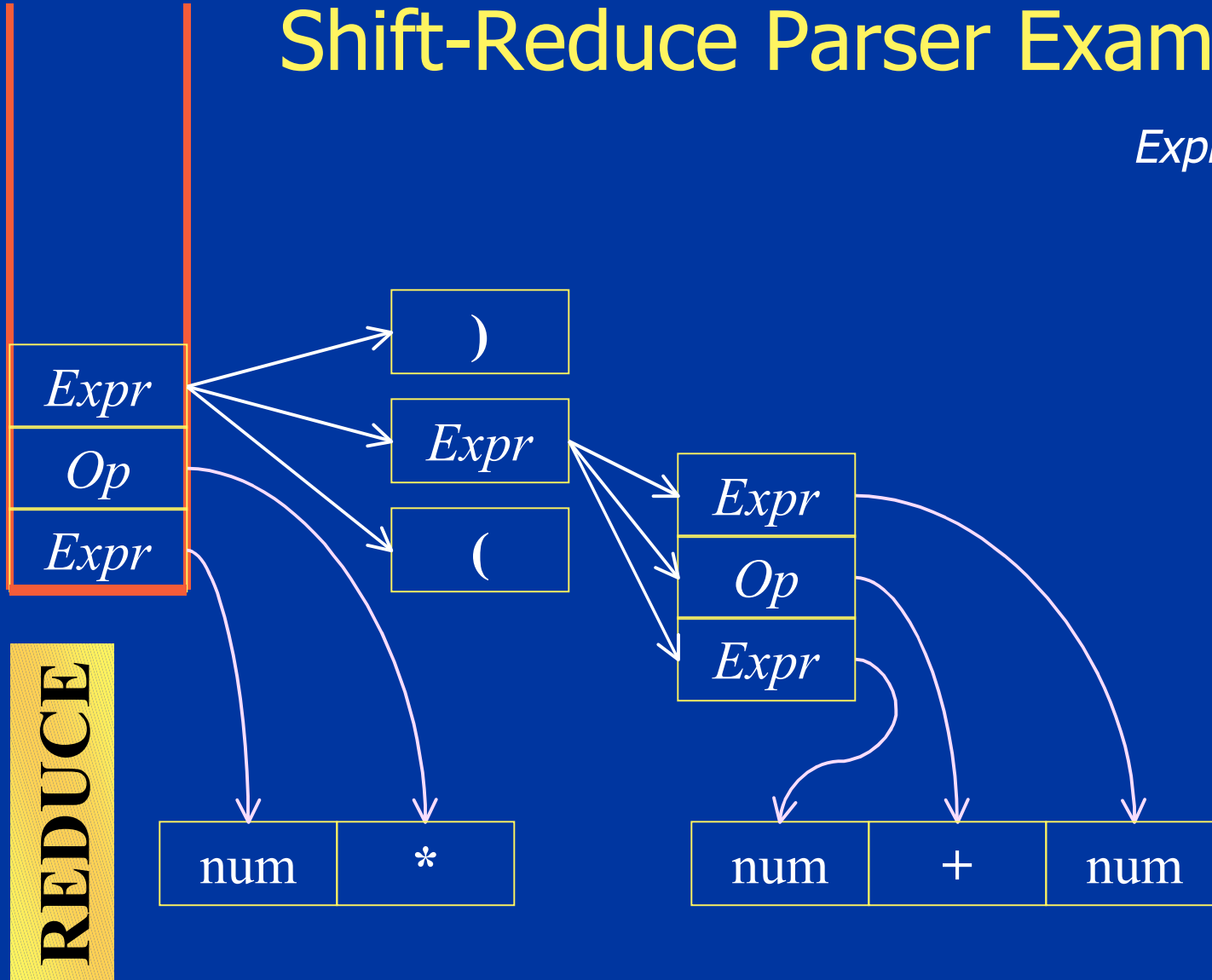
$Expr \rightarrow - Expr$

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$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

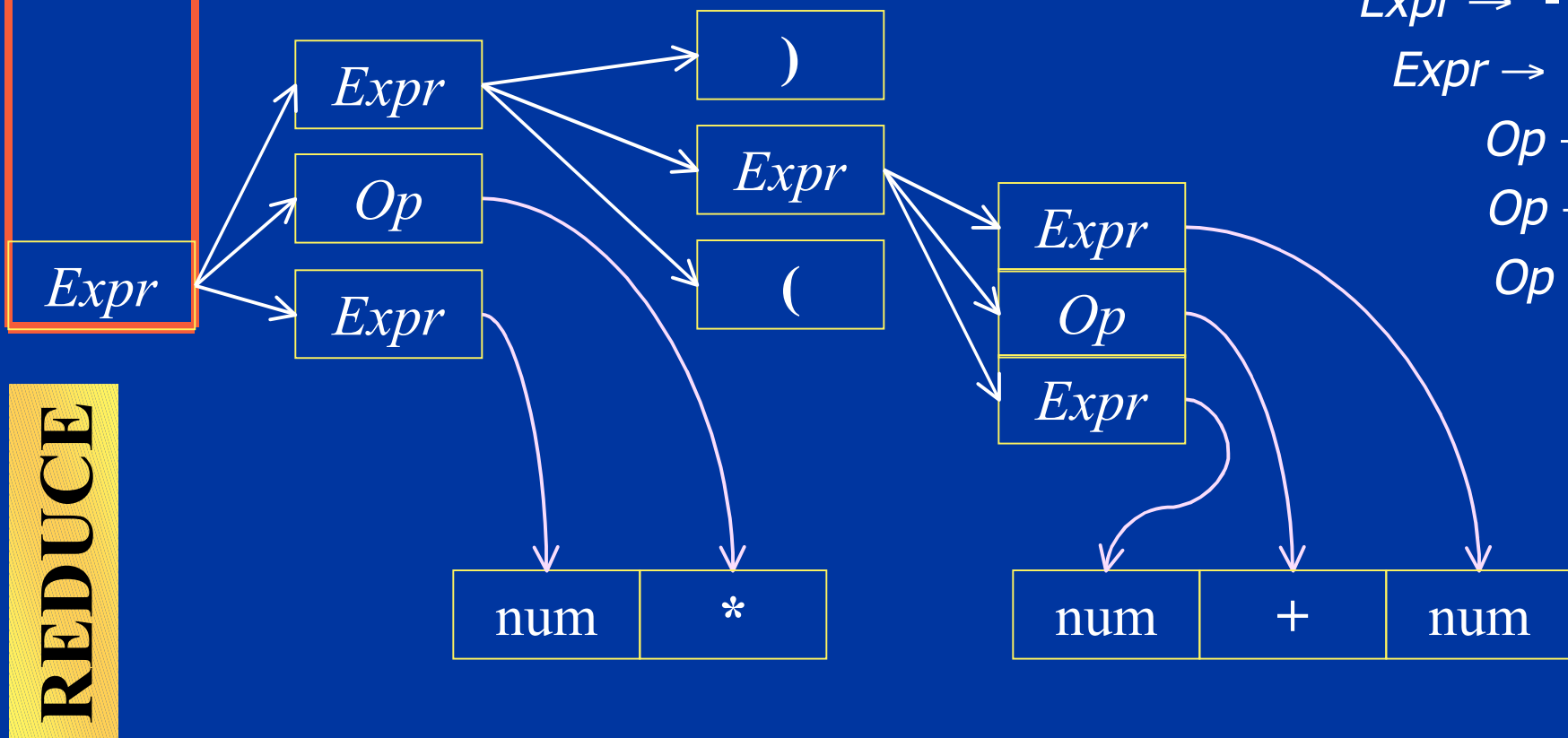
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift-Reduce Parser Example

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

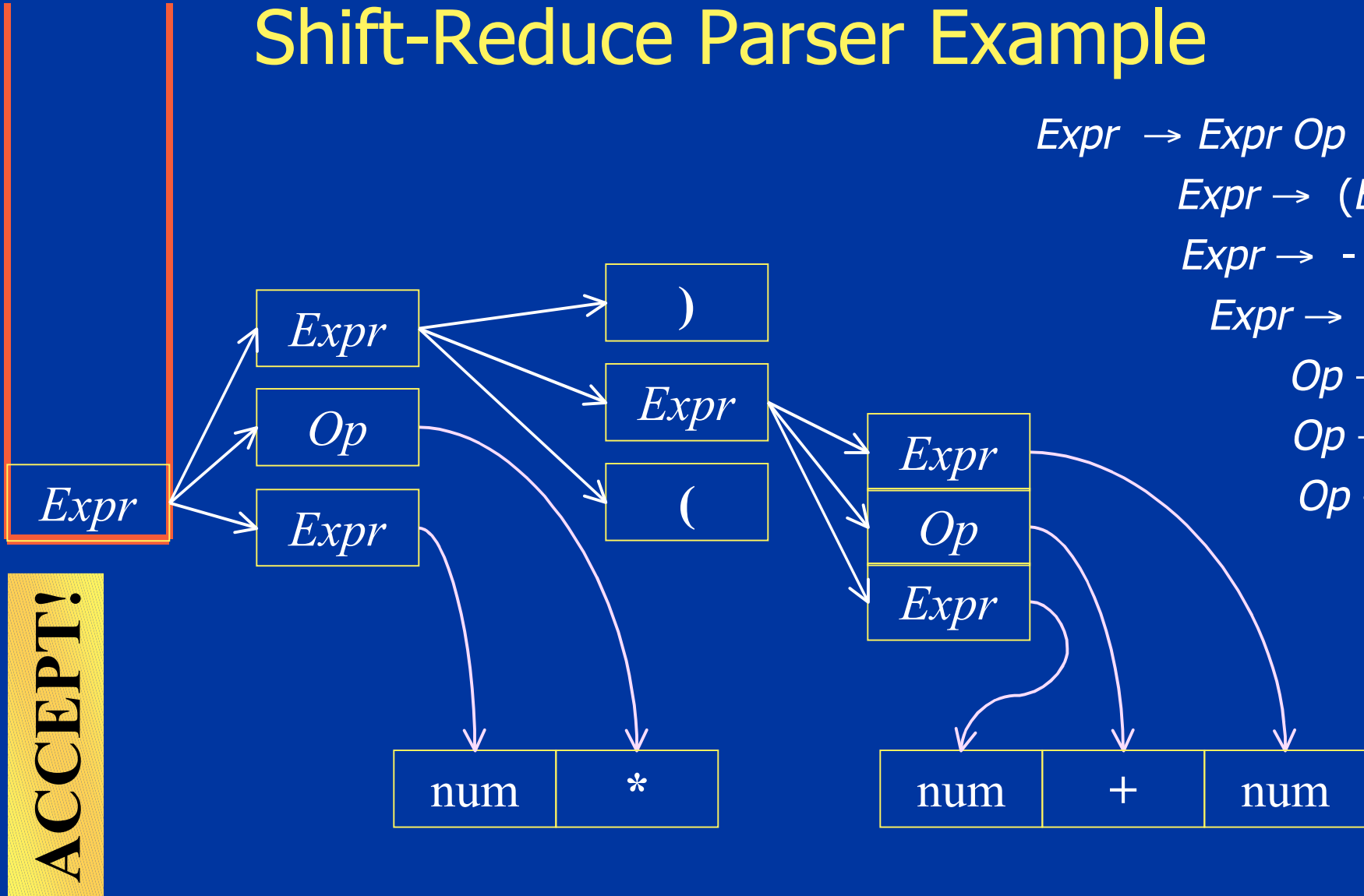
$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Basic Idea

- Goal: reconstruct parse tree for input string
- Read input from left to right
- Build tree in a bottom-up fashion
- Use stack to hold pending sequences of terminals and nonterminals

Potential Conflicts

- Reduce/Reduce Conflict
 - Top of the stack may match RHS of multiple productions
 - Which production to use in the reduction?
- Shift/Reduce Conflict
 - Stack may match RHS of production
 - But that may not be the right match
 - May need to shift an input and later find a different reduction

Conflicts

•Original Grammar

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow - Expr$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

•New Grammar

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Conflicts

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

num	-	num
-----	---	-----

Conflicts

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

SHIFT

num	-	num
-----	---	-----



Conflicts

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



SHIFT



Conflicts

$Expr \rightarrow Expr Op Expr$
 $Expr \rightarrow Expr - Expr$
 $Expr \rightarrow (Expr)$
 $Expr \rightarrow Expr -$
 $Expr \rightarrow num$
 $Op \rightarrow +$
 $Op \rightarrow -$
 $Op \rightarrow *$

Expr

REDUCE

num

-

num

Conflicts

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

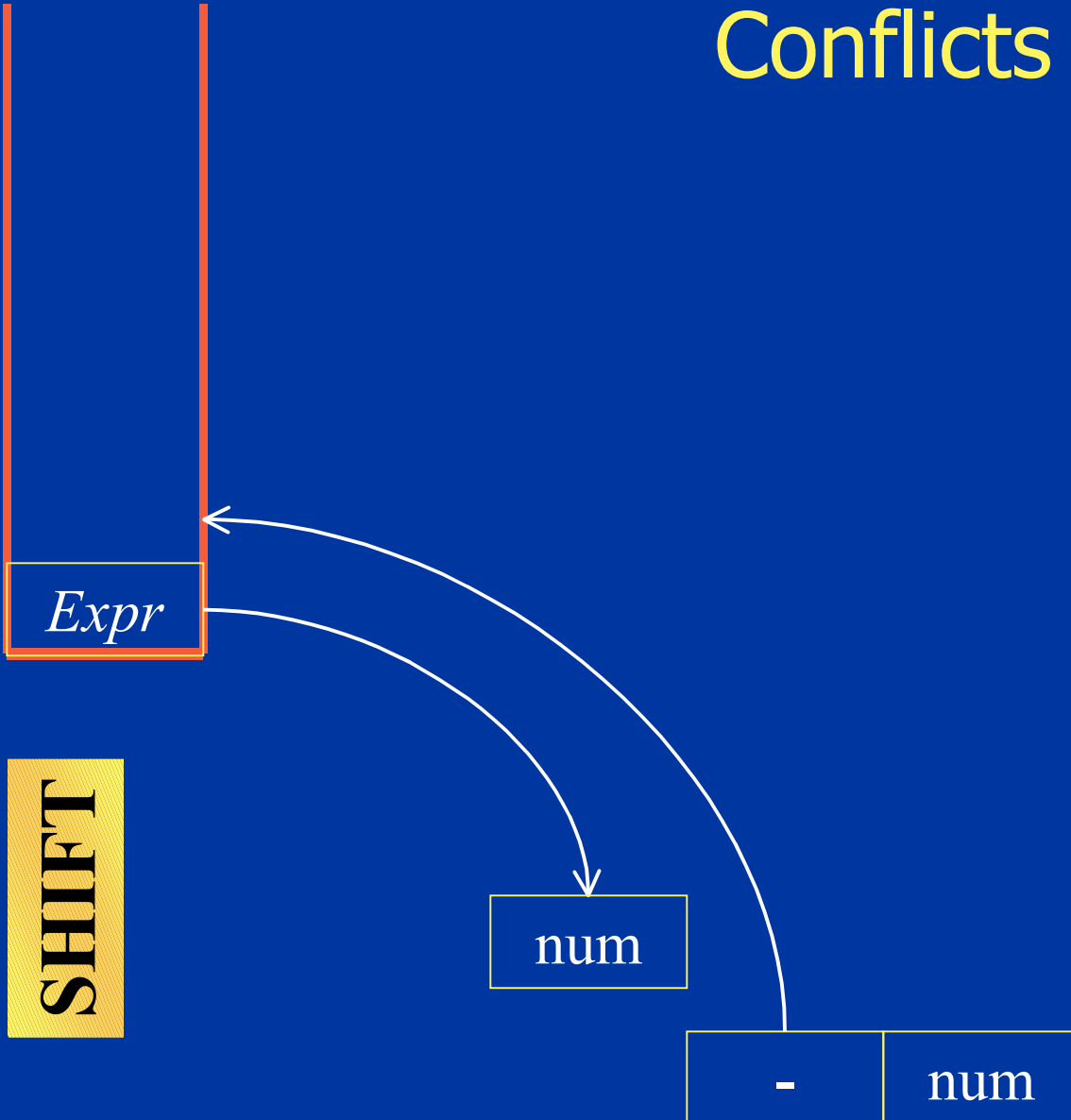
$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Conflicts

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

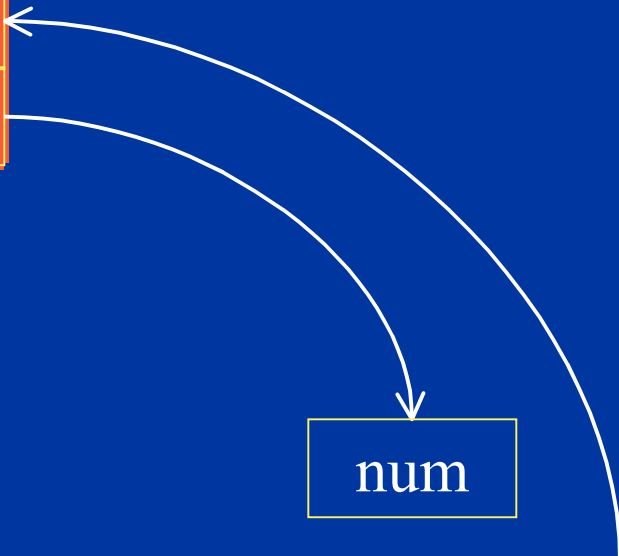
$Op \rightarrow *$



SHIFT

num

num



Shift/Reduce/Reduce Conflict

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

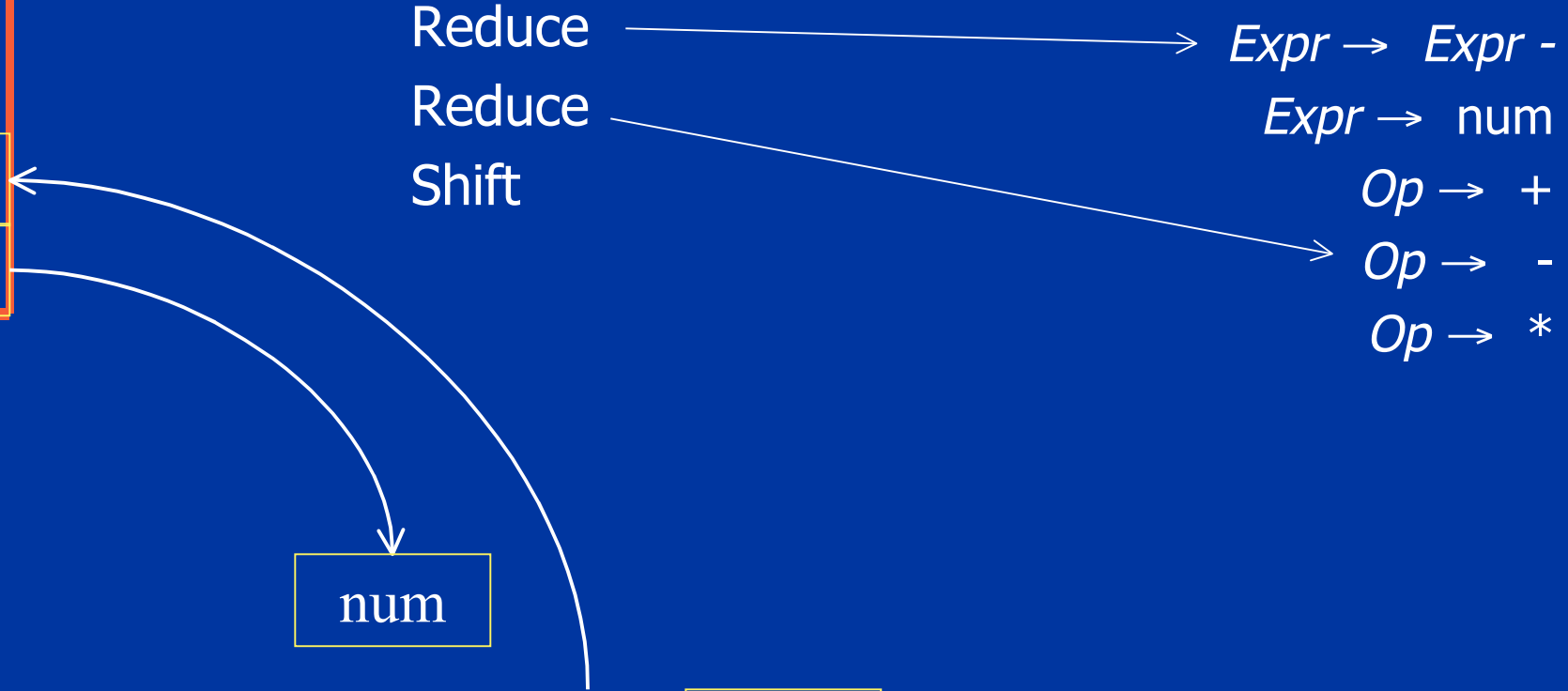
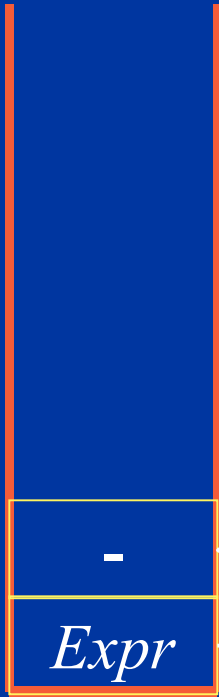
$Op \rightarrow *$

Options:

Reduce

Reduce

Shift



Shift/Reduce/Reduce Conflict

What Happens if
Choose

Reduce

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

-
Expr

REDUCE

num

num

Shift/Reduce/Reduce Conflict

What Happens if
Choose
Reduce

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

$Expr$

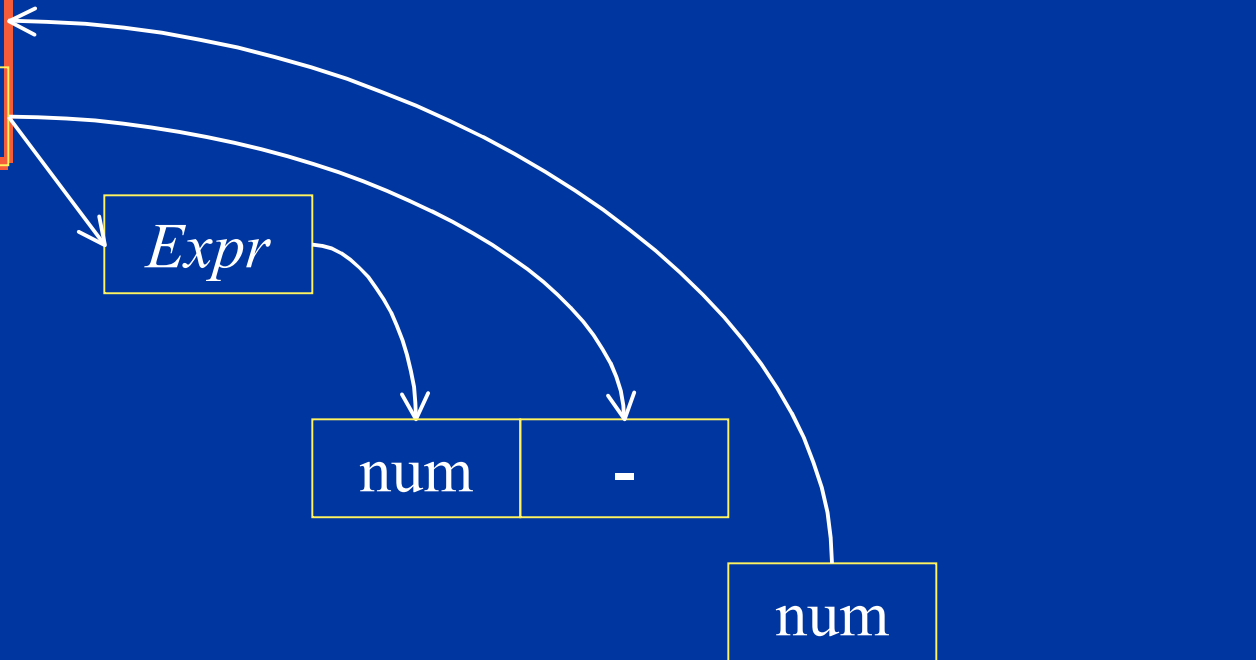
$Expr$

num

-

num

SHIFT



Shift/Reduce/Reduce Conflict

What Happens if
Choose

Reduce

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

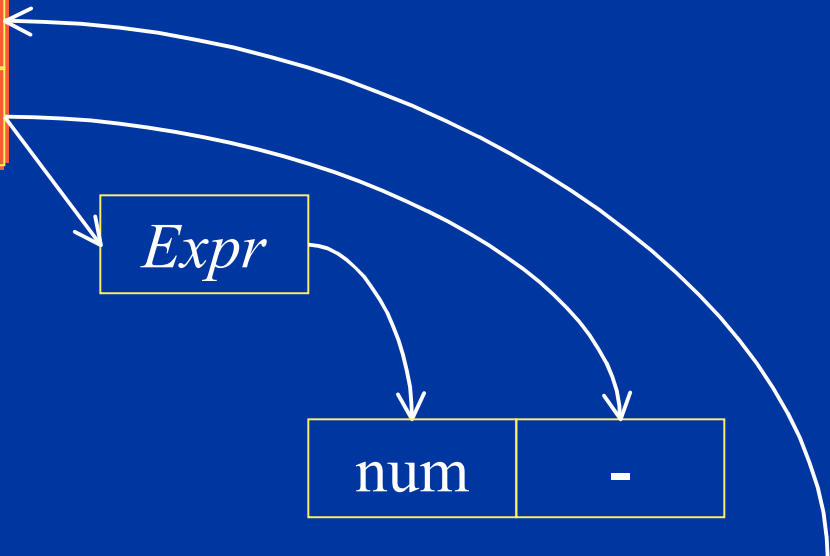
$Op \rightarrow *$

num
Expr

Expr

num -

SHIFT



Shift/Reduce/Reduce Conflict

What Happens if
Choose
Reduce

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

$Expr$

$Expr$

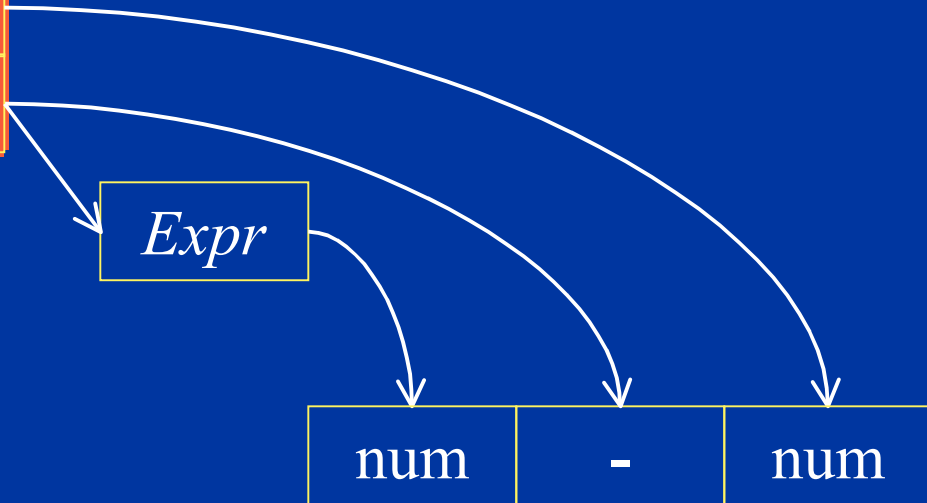
$Expr$

num

-

num

REDUCE



Shift/Reduce/Reduce Conflict

What Happens if
Choose
Reduce

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

$Expr$

$Expr$

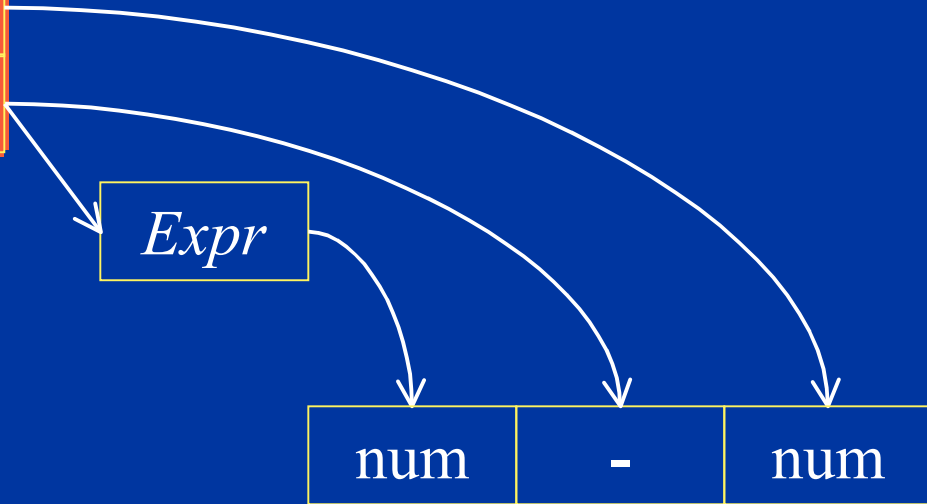
$Expr$

num

-

num

FAILS!



Shift/Reduce/Reduce Conflict

Both of These
Actions Work



Reduce
Shift

num

num

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

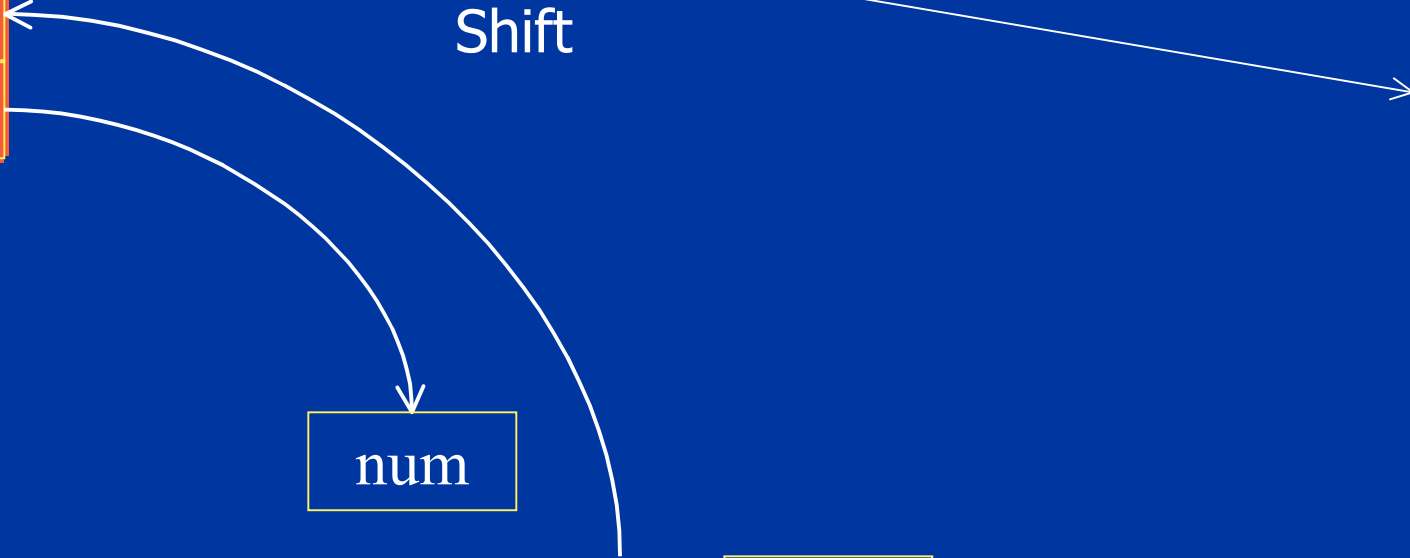
$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



Shift/Reduce/Reduce Conflict

What Happens if
Choose

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

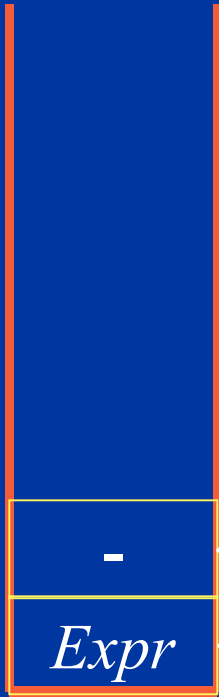
$Expr \rightarrow Expr -$

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$Op \rightarrow -$

$Op \rightarrow *$



Reduce



num

num



Shift/Reduce/Reduce Conflict

What Happens if
Choose

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

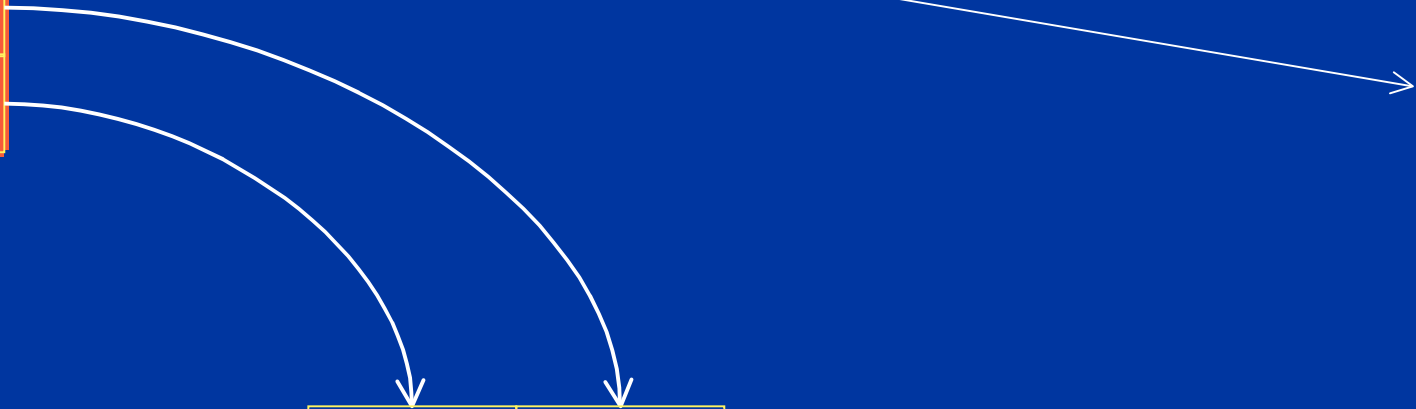
Reduce

Op
Expr

REDUCE

num -

num



Shift/Reduce/Reduce Conflict

What Happens if
Choose

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

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$Op \rightarrow *$



Reduce



SHIFT

Shift/Reduce/Reduce Conflict

What Happens if
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$Expr \rightarrow Expr -$

$Expr \rightarrow num$

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$Op \rightarrow -$

$Op \rightarrow *$

Expr
Op
Expr

Reduce

REDUCE

num - num

Shift/Reduce/Reduce Conflict

What Happens if
Choose

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Expr



Reduce



REDUCE

Shift/Reduce/Reduce Conflict

What Happens if
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$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

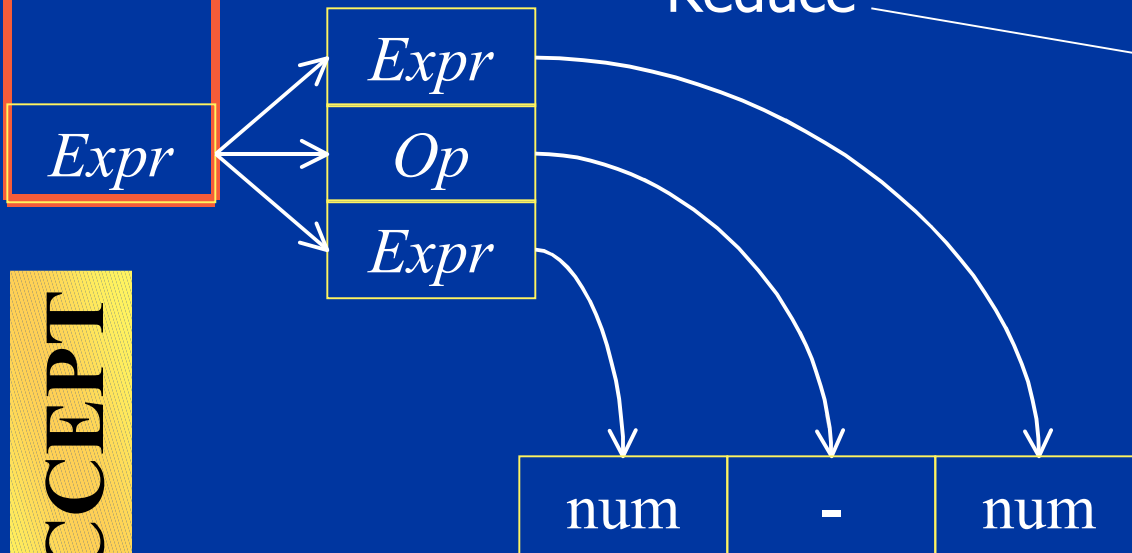
$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

Reduce

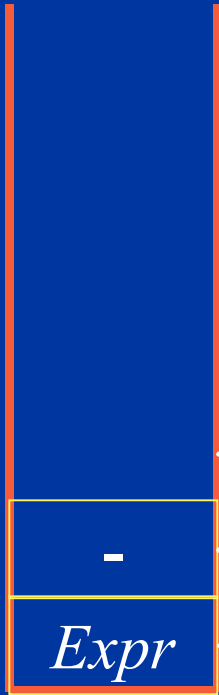


ACCEPT

Conflicts

What Happens if
Choose

Shift



SHIFT

num

num

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$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

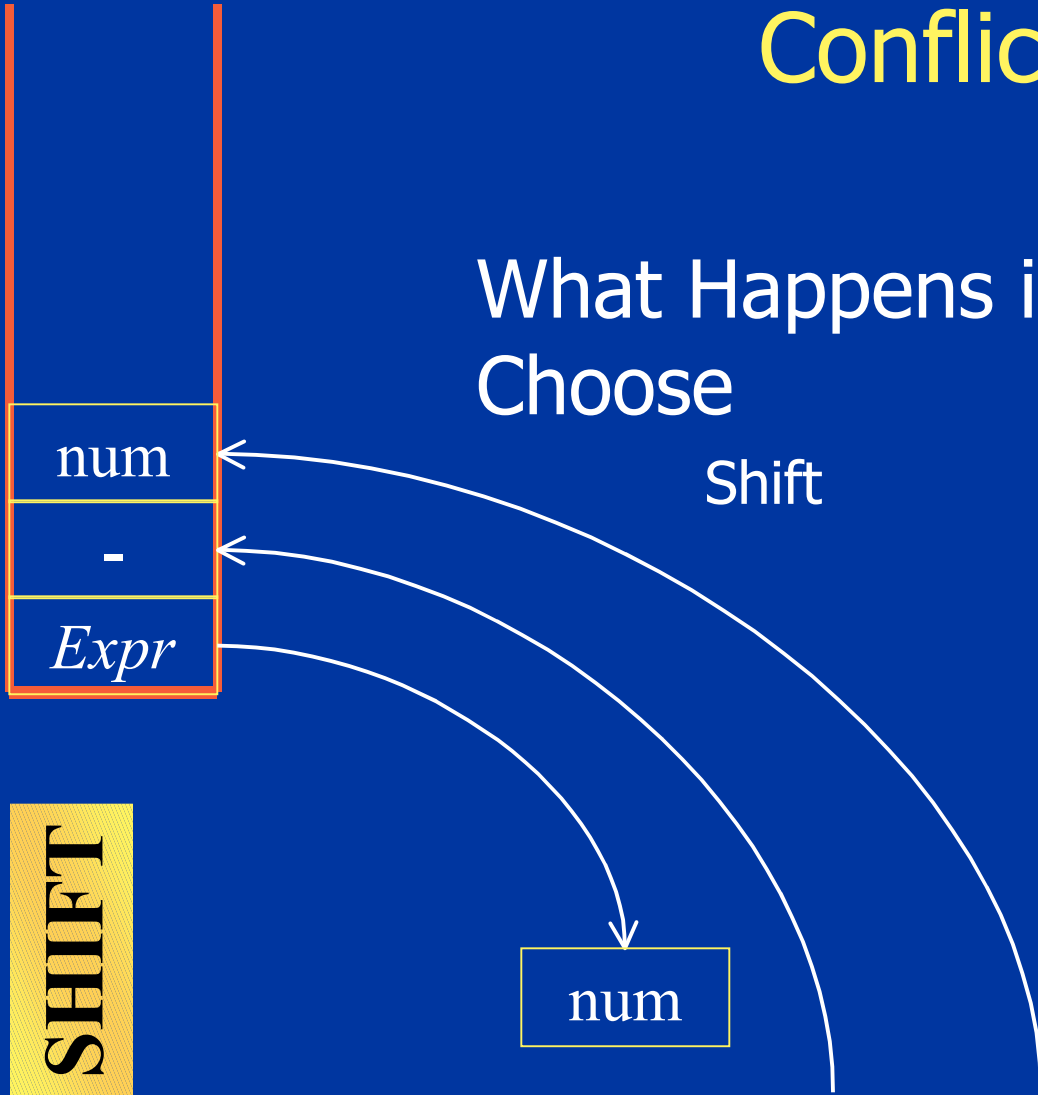
$Op \rightarrow -$

$Op \rightarrow *$

Conflicts

What Happens if
Choose

Shift



$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

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$Expr \rightarrow num$

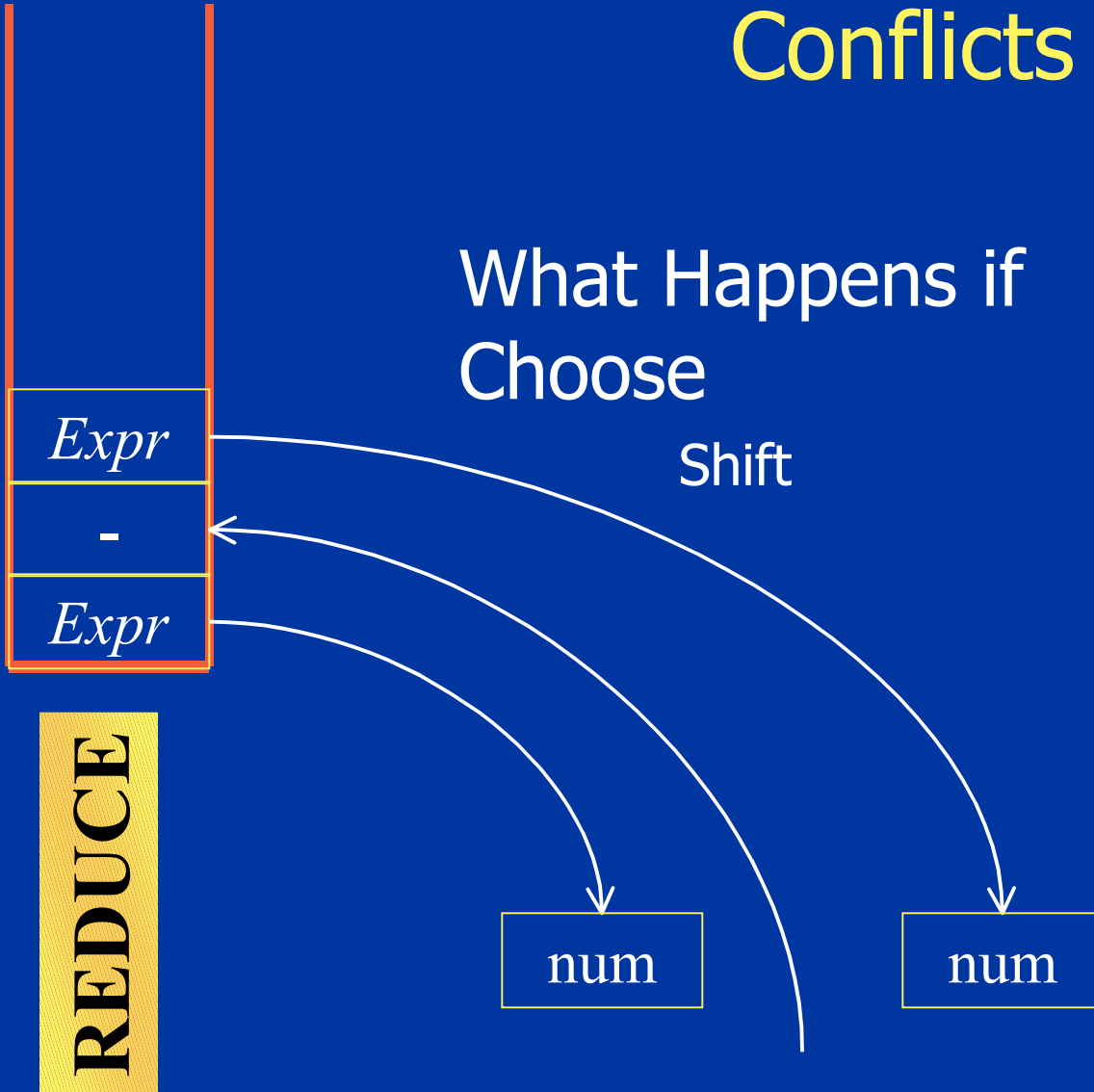
$Op \rightarrow +$

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Conflicts

What Happens if
Choose



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$Expr \rightarrow Expr - Expr$

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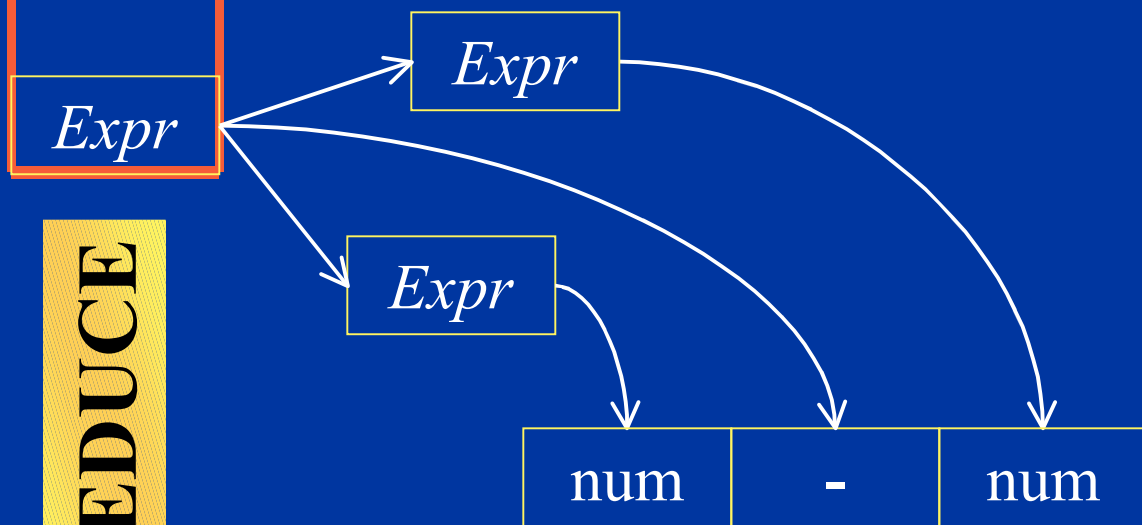
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$Op \rightarrow *$

Conflicts

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Shift



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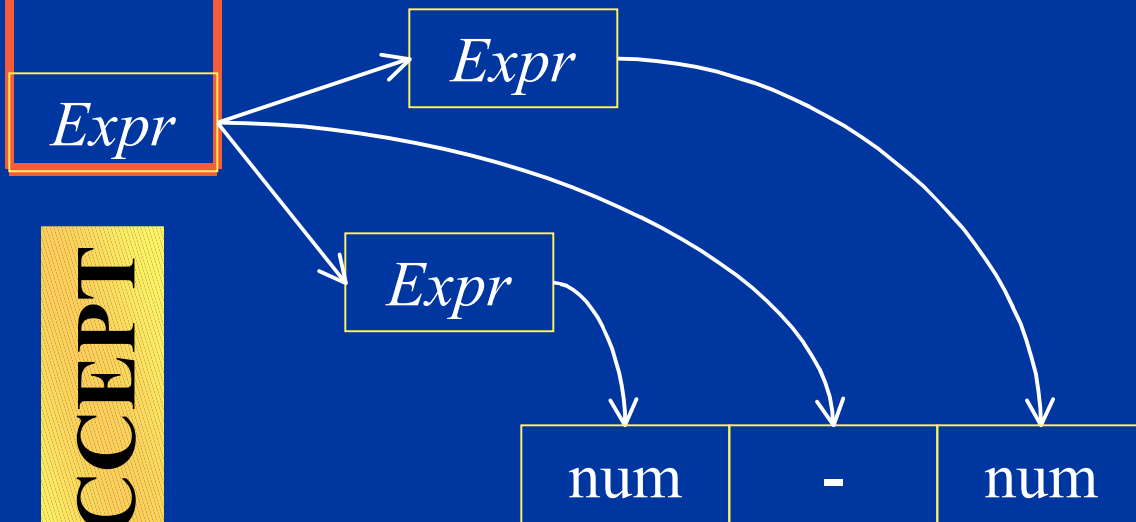
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Conflicts

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$Op \rightarrow -$

$Op \rightarrow *$

Shift/Reduce/Reduce Conflict

This Shift/Reduce Conflict
Reflects Ambiguity in
Grammar

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

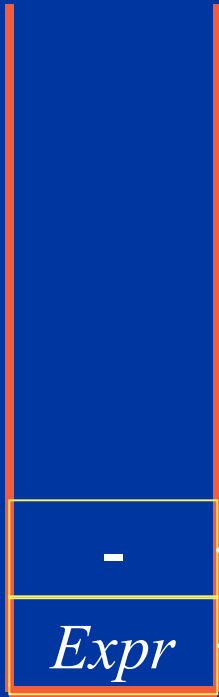
$Expr \rightarrow Expr -$

$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$



num

num

Shift/Reduce/Reduce Conflict

This Shift/Reduce Conflict Reflects Ambiguity in Grammar

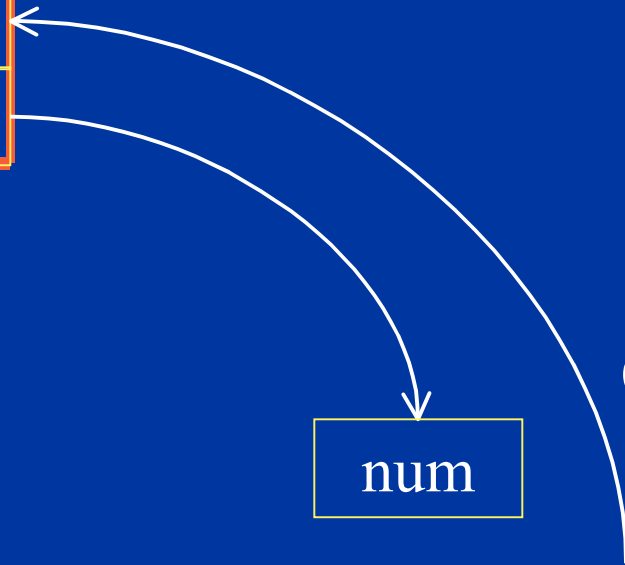
$Expr \rightarrow Expr Op Expr$
 ~~$Expr \rightarrow Expr - Expr$~~
 $Expr \rightarrow (Expr)$
 $Expr \rightarrow Expr -$
 $Expr \rightarrow num$
 $Op \rightarrow +$
 $Op \rightarrow -$
 $Op \rightarrow *$



Eliminate by Hacking Grammar

num

num



Shift/Reduce/Reduce Conflict

$Expr \rightarrow Expr Op Expr$

$Expr \rightarrow Expr - Expr$

$Expr \rightarrow (Expr)$

$Expr \rightarrow Expr -$

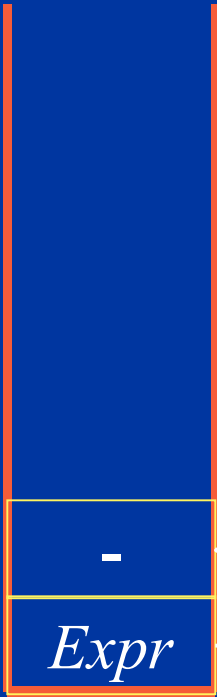
$Expr \rightarrow num$

$Op \rightarrow +$

$Op \rightarrow -$

$Op \rightarrow *$

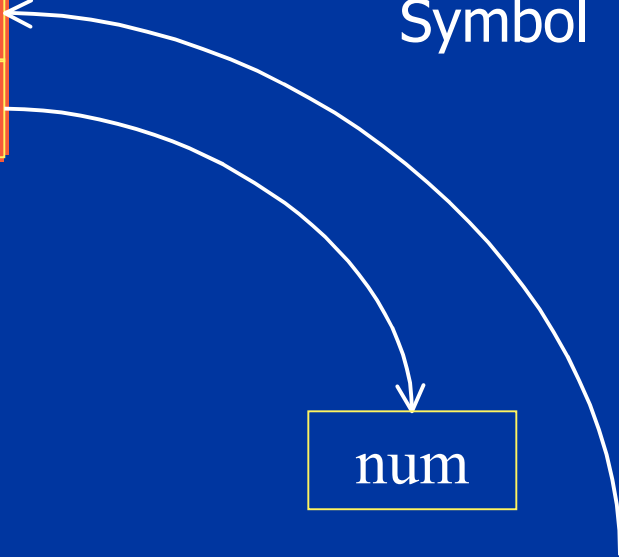
This Shift/Reduce
Conflict Can Be
Eliminated By
Lookahead of One
Symbol



Parser Generator Should
Handle It

num

num



Constructing a Parser

- We will construct version with no lookahead
- Key Decisions
 - Shift or Reduce
 - Which Production to Reduce
- Basic Idea
 - Build a DFA to control shift and reduce actions
 - In effect, convert grammar to pushdown automaton
 - Encode finite state control in parse table

Parser State

- Input Token Sequence (\$ for end of input)
- Current State from Finite State Automaton
- Two Stacks
 - State Stack (implements finite state automaton)
 - Symbol Stack (terminals from input and nonterminals from reductions)

Integrating Finite State Control

- Actions
 - Push Symbols and States Onto Stacks
 - Reduce According to a Given Production
 - Accept
- Selected action is a function of
 - Current input symbol
 - Current state of finite state control
- Each action specifies next state
- Implement control using parse table

Parse Tables

		ACTION		Goto
State	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

- Implements finite state control
- At each step, look up
 - Table[top of state stack] [input symbol]
- Then carry out the action

Parse Table Example

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

(0)

s0

X

Parser Tables

	ACTION			Goto
State	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

- Shift to s_n
 - Push input token into the symbol stack
 - Push s_n into state stack
 - Advance to next input symbol

Parser Tables

		ACTION		Goto
State	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

- Reduce (n)
 - Pop both stacks as many times as the number of symbols on the RHS of rule n
 - Push LHS of rule n into symbol stack

Parser Tables

		ACTION			Goto
State	()	\$	X	
s0	shift to s2	error	error	goto s1	
s1	error	error	accept		
s2	shift to s2	shift to s5	error	goto s3	
s3	error	shift to s4	error		
s4	reduce (2)	reduce (2)	reduce (2)		
s5	reduce (2)	reduce (2)	reduce (2)		

- Reduce (n) (continued)
 - Look up
 - Table[top of the state stack][top of symbol stack]
 - Push that state (in goto part of table) onto state stack

Parser Tables

	ACTION			Goto
State	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

- Accept
 - Stop parsing and report success

Parse Table In Action

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar

()\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

s0

Parse Table In Action

State	ACTION			Goto
	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar

()\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

s0

Parse Table In Action

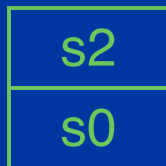
State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar



)\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Parse Table In Action

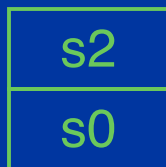
State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar



)\$

$S \rightarrow X\$$ (1)

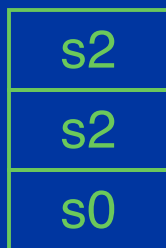
$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

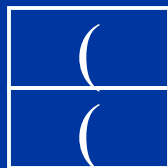
Parse Table In Action

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack



Symbol Stack



Input

))\$

Grammar

$S \rightarrow X\$$ (1)

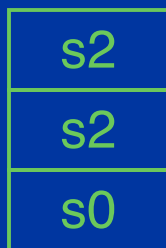
$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

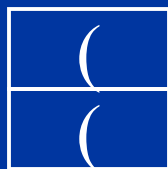
Parse Table In Action

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack



Symbol Stack



Input

))\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Parse Table In Action

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

s5
s2
s2
s0

Symbol Stack

)
(
(

Input

)\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Parse Table In Action

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

s5
s2
s2
s0

Symbol Stack

)
(
(

Input

)\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Step One: Pop Stacks

State	(ACTION		Goto
)	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

s5
s2
s2
s0

Symbol Stack

)
(
(

Input

)\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Step One: Pop Stacks

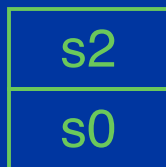
State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar



)\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Step Two: Push Nonterminal

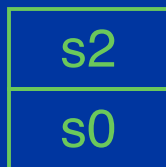
State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar



)\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Step Two: Push Nonterminal

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar

s2
s0

X
(

)\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Step Three: Use Goto, Push New State

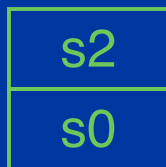
		ACTION		Goto
State	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar



)\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Step Three: Use Goto, Push New State

		ACTION		Goto
State	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

s3
s2
s0

Symbol Stack

X
(

Input

)\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Parse Table In Action

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

s3
s2
s0

Symbol Stack

X
(

Input

)\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Parse Table In Action

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

s4
s3
s2
s0

Symbol Stack

)
X
(

Input

\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Parse Table In Action

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

s4
s3
s2
s0

Symbol Stack

)
X
(

Input

\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Step One: Pop Stacks

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

s4
s3
s2
s0

Symbol Stack

)
X
(

Input

\$

Grammar

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Step One: Pop Stacks

		ACTION		Goto
State	()	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar

\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

s0

Step Two: Push Nonterminal

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar

\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

s0

Step Two: Push Nonterminal

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar

\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

s0

X

Step Three: Use Goto, Push New State

State	(ACTION		Goto
)	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar

\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

s0

X

Step Three: Use Goto, Push New State

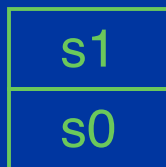
State	(ACTION		Goto
)	\$	X
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

Grammar



\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)

Accept the String!

State	ACTION			Goto
	()	\$	
s0	shift to s2	error	error	goto s1
s1	error	error	accept	
s2	shift to s2	shift to s5	error	goto s3
s3	error	shift to s4	error	
s4	reduce (2)	reduce (2)	reduce (2)	
s5	reduce (2)	reduce (2)	reduce (2)	

State Stack

Symbol Stack

Input

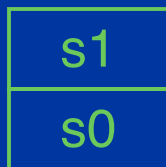
Grammar

\$

$S \rightarrow X\$$ (1)

$X \rightarrow (X)$ (2)

$X \rightarrow ()$ (3)



Key Concepts

- Pushdown automaton for parsing
 - Stack, Finite state control
 - Parse actions: shift, reduce, accept
- Parse table for controlling parser actions
 - Indexed by parser state and input symbol
 - Entries specify action and next state
 - Use state stack to help control
- Parse tree construction
 - Reads input from left to right
 - Bottom-up construction of parse tree