Studying Algorithms

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Algorithms

General recipes for solving problems...
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General recipes for solving problems **not** specific of any language or platform.
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We specify the input, desired output, and the steps to get from one to the other.
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**Coq au Vin (partial recipe)**

*While* the liquid is boiling, in a small bowl, blend the 3 tablespoons flour and 2 tablespoons softened butter into a smooth paste.

Beat the flour/butter mixture into the approximately 2 cups hot cooking liquid with a whisk.

Simmer and stir for a minute or two *until* the sauce has thickened.

*If* the sauce doesn’t thicken right away...

Sequence, Alternation, and Repetition.
Algorithms

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- Searching
- Sorting
- Data Structures
- Graphs
- Trees
- Dynamic Programming
- and more . . .
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Challenges:
  • Correctness
  • Efficiency
  • Applicability
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Does it work? I.e., Is the output correct for all inputs, all instances, and all edge cases?

Also, does it halt?

And can you prove it?
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• with a reasonable amount of effort?
• using a reasonable amount of resources?
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There are many algorithms.
There are many data structures.
Did you choose the right one at the right time in the right place for the right use case?

Or were you in the right place at the wrong time?
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Q: How can we characterize algorithms in a manner that’s not specific of any language or platform?
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A: Growth functions.

Examples:

- O(n) “Order n” or “Big-oh of n”
- O(n²) “Order n squared” or “Big-oh of n squared”
- O(log₂ n) “Order log to the base two of n” or . . .
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Q: How can we characterize algorithms in a manner that’s not specific of any language or platform?

A: Growth functions let us characterize how the time/effort/space required to execute the algorithm grows as the size of the input grows.

Think of this as “complexity”.

We’re concerned with the measures of effort/complexity needed to correctly solve a problem.

We’re also concerned with how those measures change proportionally with the size of the input. I.e., how does the effort scale or grow with the input? What is its “order of growth”? 
Characterizing algorithms in terms of complexity

**Good news.** The set of functions

\[ 1, \log n, n, n \log n, n^2, n^3, \text{ and } 2^n \]

suffices to describe the order of growth of most common algorithms.

*from Robert Sedgewick and Kevin Wayne’s Princeton Algorithms course notes*
Characterizing algorithms in terms of complexity

### Common order-of-growth classifications

<table>
<thead>
<tr>
<th>order of growth</th>
<th>name</th>
<th>typical code framework</th>
<th>description</th>
<th>example</th>
<th>(T(2n) / T(n))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>constant</td>
<td>(a = b + c;)</td>
<td>statement</td>
<td>add two numbers</td>
<td>1</td>
</tr>
<tr>
<td>(\log n)</td>
<td>logarithmic</td>
<td>while ((n &gt; 1)) { (n = n/2; \ldots ) }</td>
<td>divide in half</td>
<td>binary search</td>
<td>(\sim 1)</td>
</tr>
<tr>
<td>(n)</td>
<td>linear</td>
<td>for ((\text{int } i = 0; i &lt; n; i++)) { \ldots }</td>
<td>single loop</td>
<td>find the maximum</td>
<td>2</td>
</tr>
<tr>
<td>(n \log n)</td>
<td>linearithmic</td>
<td>mergesort.</td>
<td>divide and conquer</td>
<td>mergesort</td>
<td>(\sim 2)</td>
</tr>
<tr>
<td>(n^2)</td>
<td>quadratic</td>
<td>for ((\text{int } i = 0; i &lt; n; i++)) { \ldots } for ((\text{int } j = 0; j &lt; n; j++)) { \ldots }</td>
<td>double loop</td>
<td>check all pairs</td>
<td>4</td>
</tr>
<tr>
<td>(n^3)</td>
<td>cubic</td>
<td>for ((\text{int } i = 0; i &lt; n; i++)) { \ldots } for ((\text{int } j = 0; j &lt; n; j++)) { \ldots } for ((\text{int } k = 0; k &lt; n; k++)) { \ldots }</td>
<td>triple loop</td>
<td>check all triples</td>
<td>8</td>
</tr>
<tr>
<td>(2^n)</td>
<td>exponential</td>
<td>combinatorial search</td>
<td>exhaustive search</td>
<td>check all subsets</td>
<td>(2^n)</td>
</tr>
</tbody>
</table>
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Ponder this ...

1.2-3
What is the smallest value of $n$ such that an algorithm whose running time is $100n^2$ runs faster than an algorithm whose running time is $2^n$ on the same machine?

... and see if you can produce this graph in Desmos: