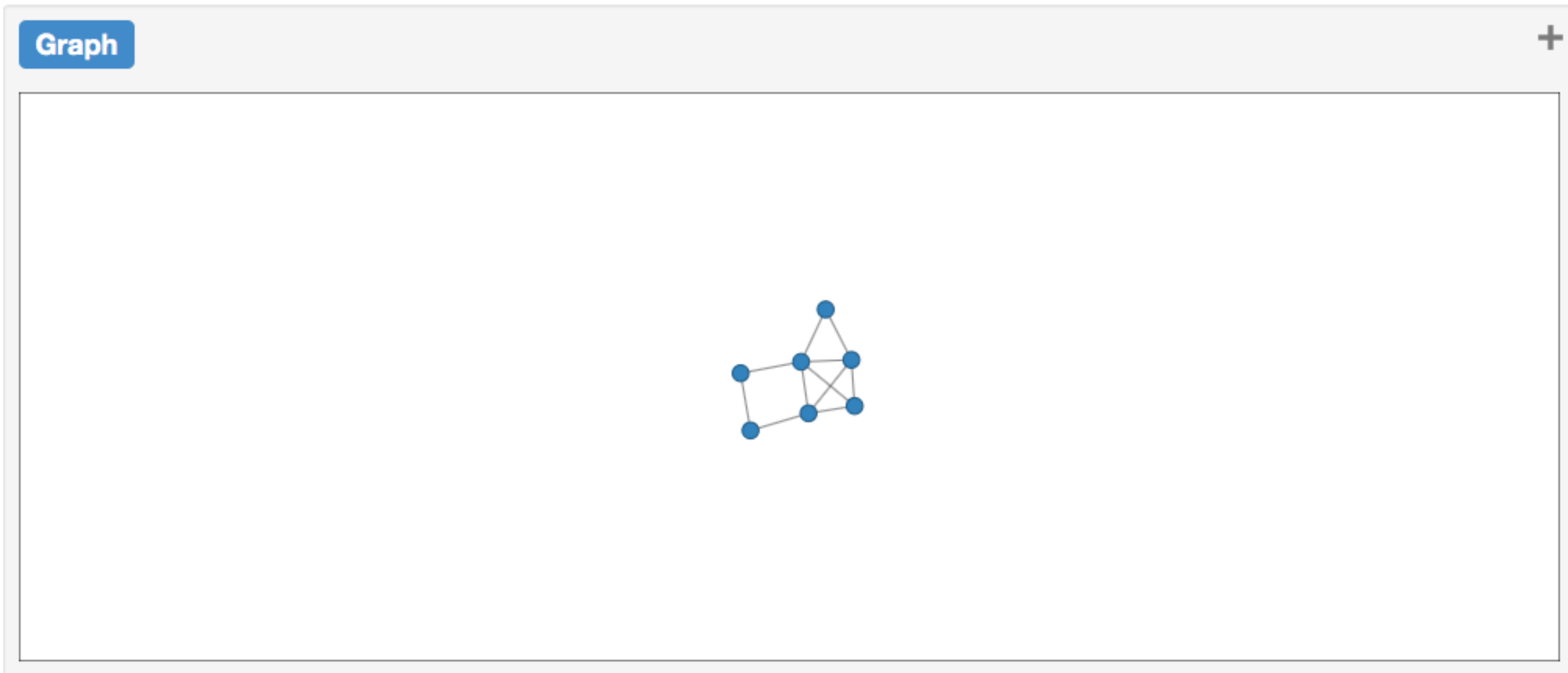
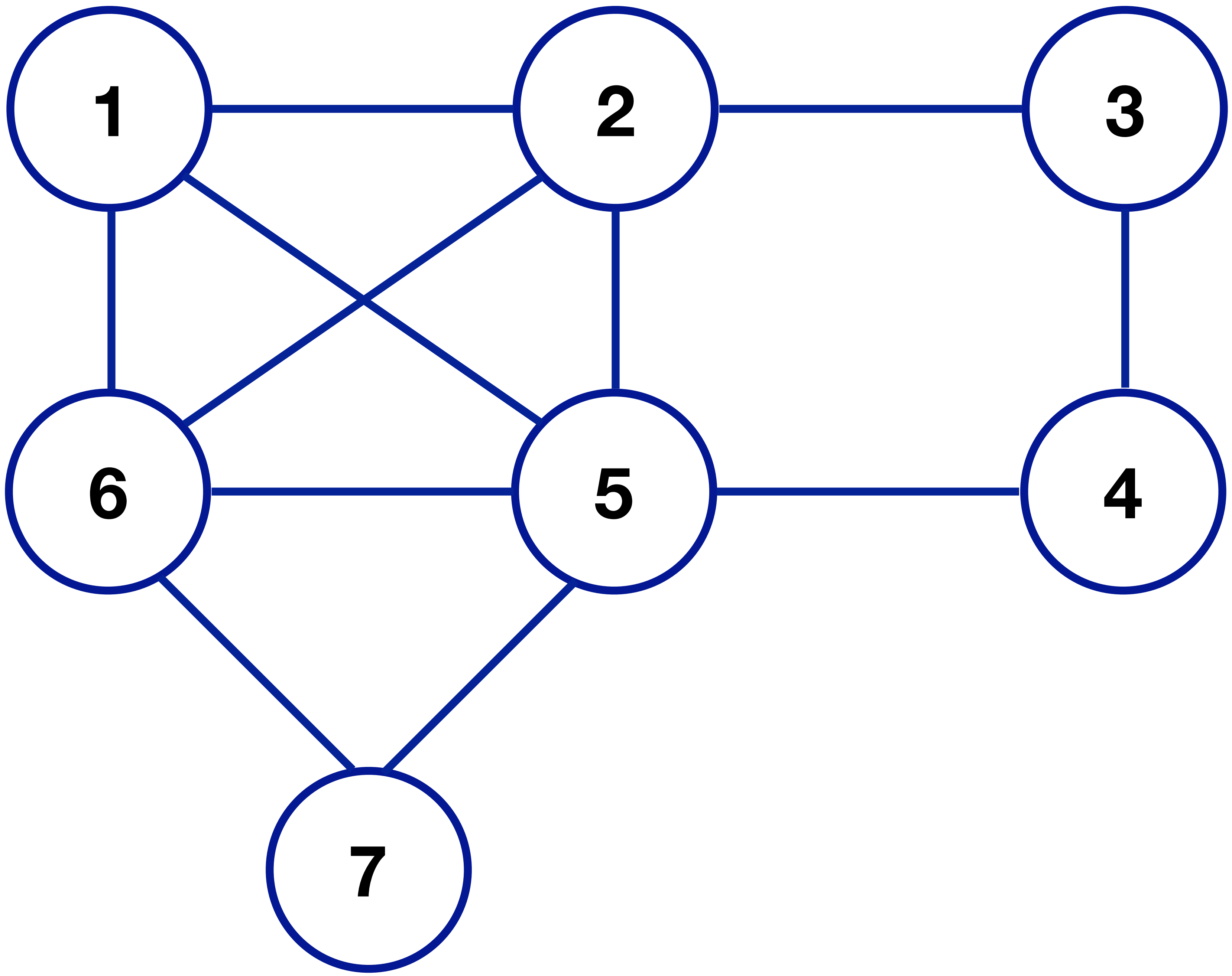
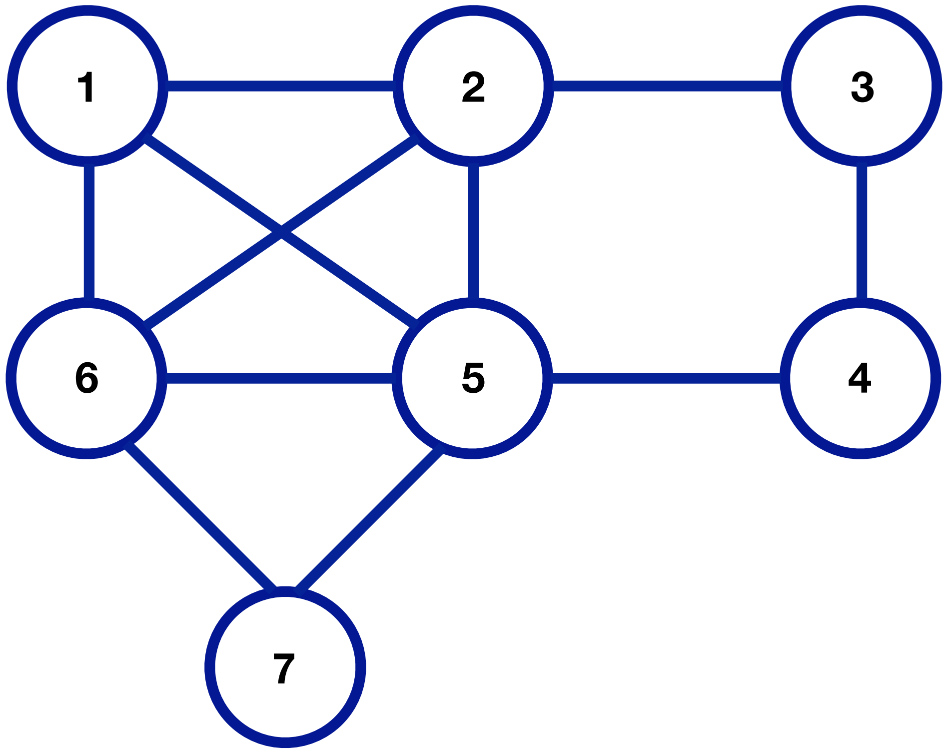
G*studio




1
2
3
6
5
4
7

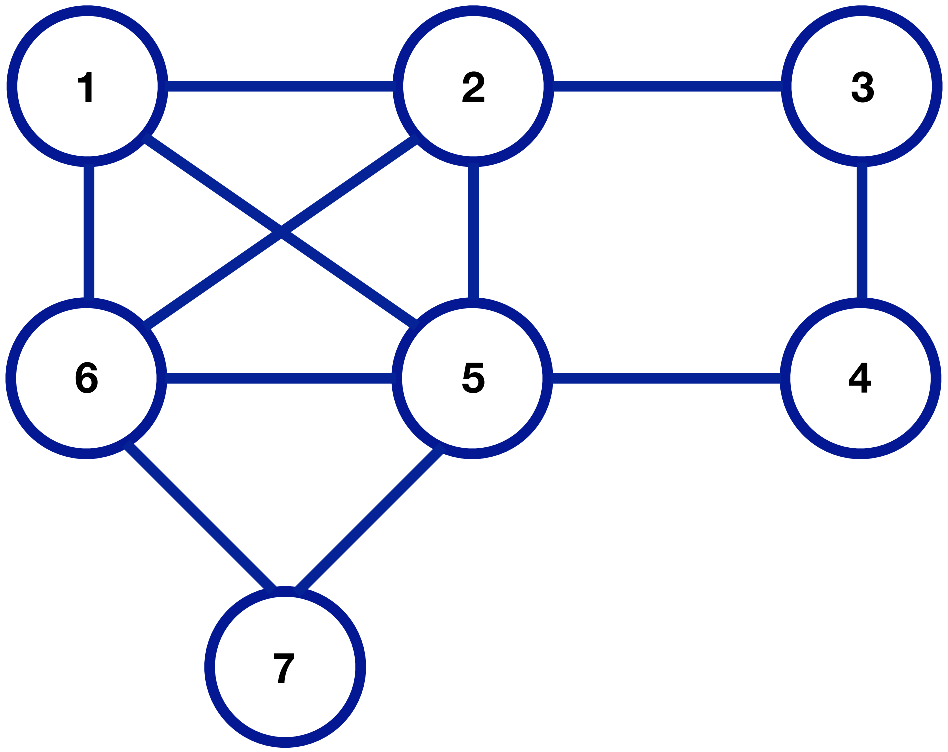
Graph . . .



as Matrix

	1	2	3	4	5	6	7
1	.	1	.	.	1	1	.
2	1	.	1	.	1	1	.
3	.	1	.	1	.	.	.
4	.	.	1	.	1	.	.
5	1	1	.	1	.	1	1
6	1	1	.	.	1	.	1
7	.	.	.	.	1	1	.

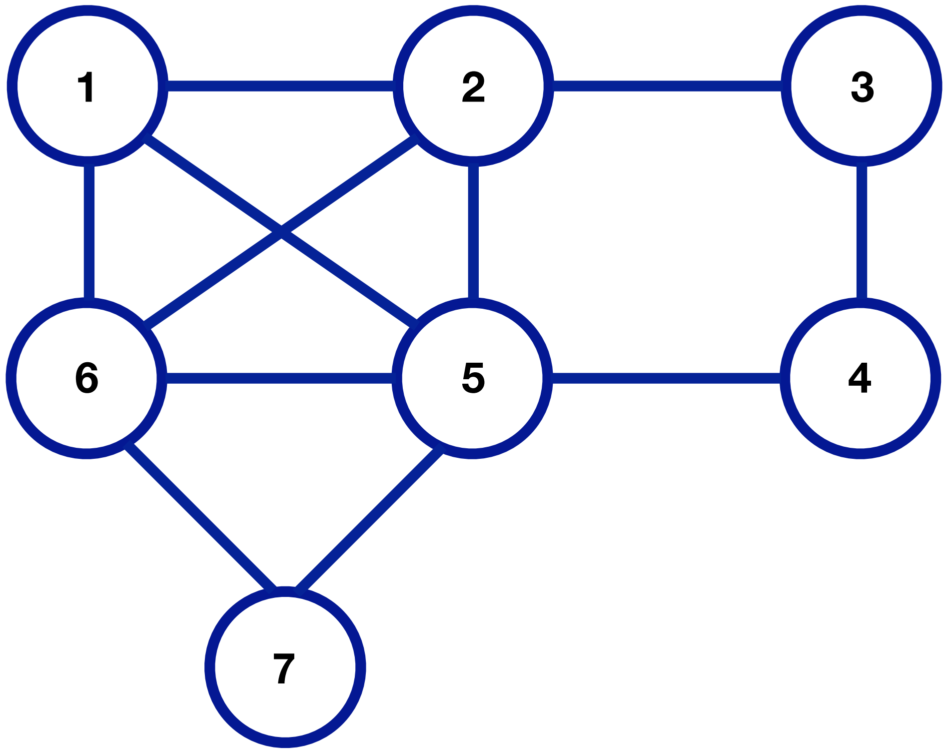
Graph . . .



as Adjacency List

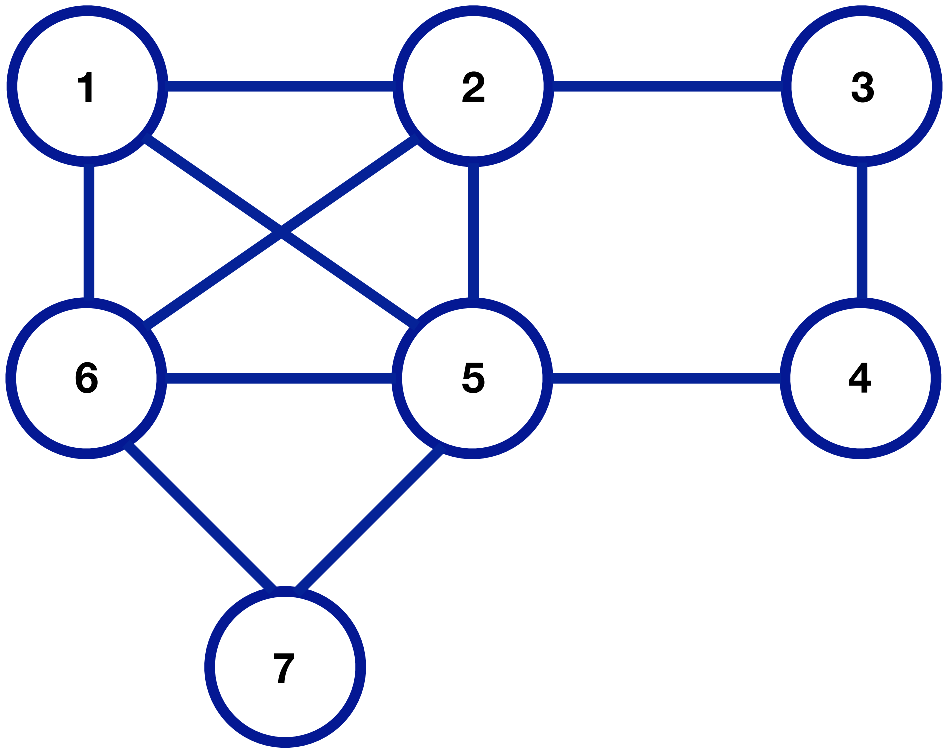
```
[1]  2  5  6
[2]  1  3  5  6
[3]  2  4
[4]  3  5
[5]  1  2  4  6  7
[6]  1  2  5  7
[7]  5  6
```

Independent Sets



An **independent set** in a graph *G* = (*V*, *E*) is a subset *V'* ⊆ *V* such that, for all *u*,*v* in *V'*, the edge {*u*,*v*} is **not** in E. (I.e., no two vertices in *V'* are adjacent.)

Independent Sets



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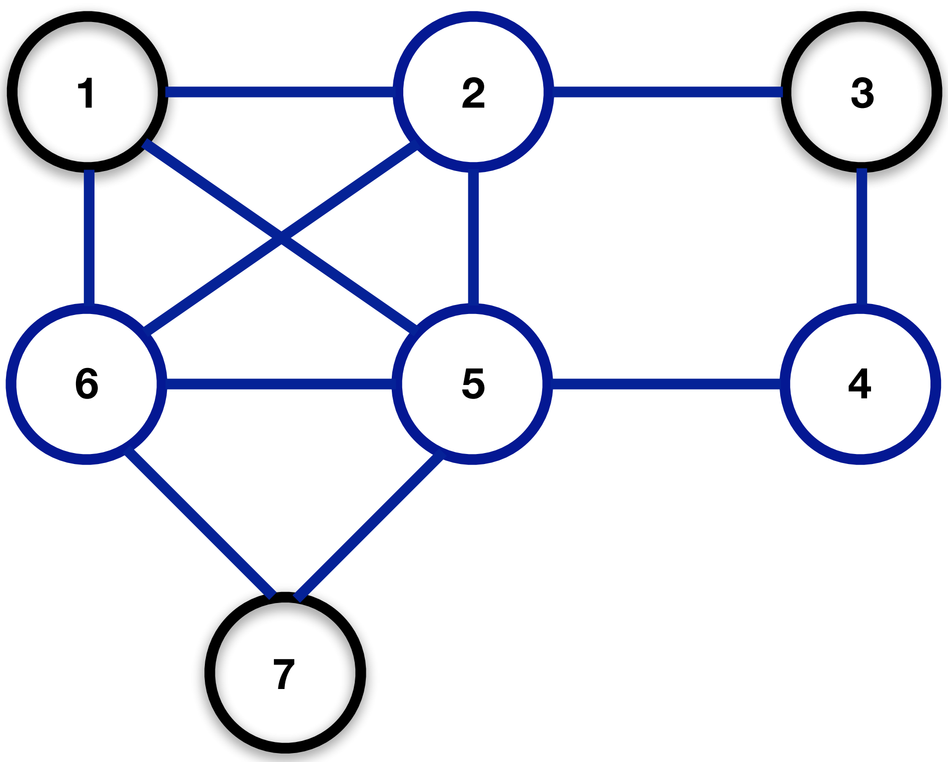
A **maximum independent set** is an independent set of the largest possible cardinality.

Can you find one?

Independent Sets



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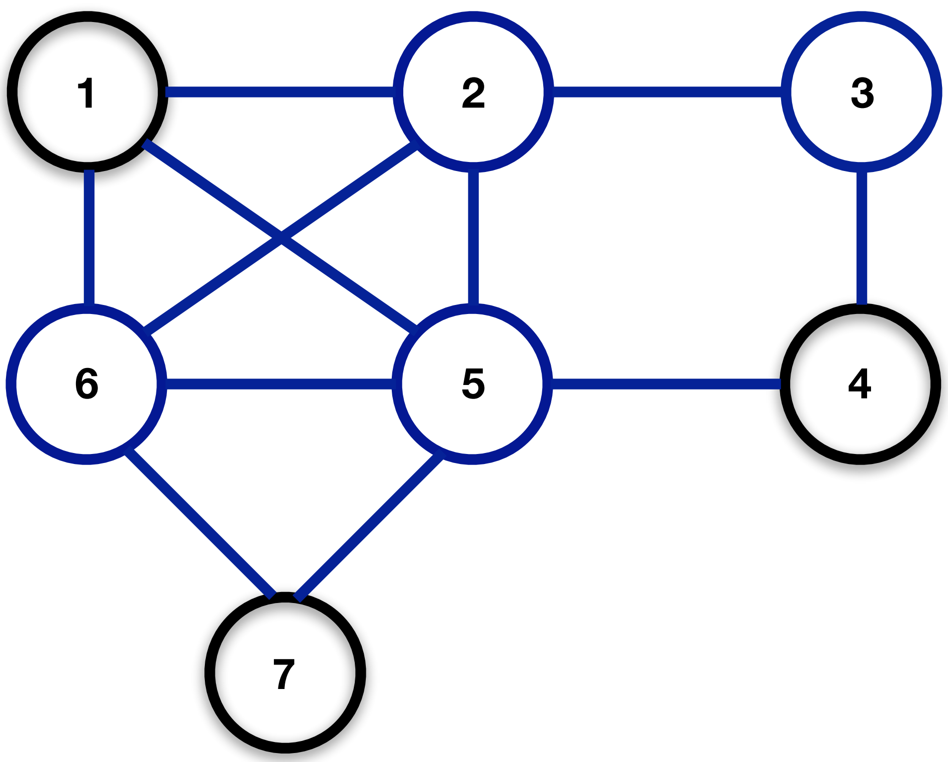
Can you find one? 1 3 7

Can you find two?

Independent Sets



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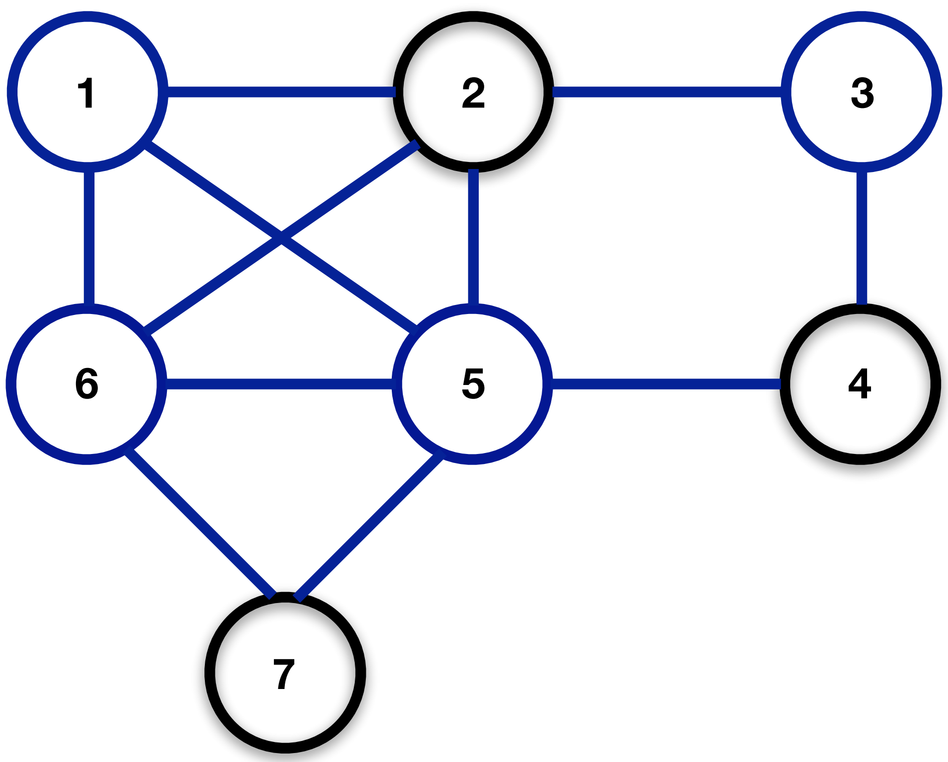
Can you find one? 1 3 7

Can you find two? 1 4 7

Can you find three?

Independent Sets



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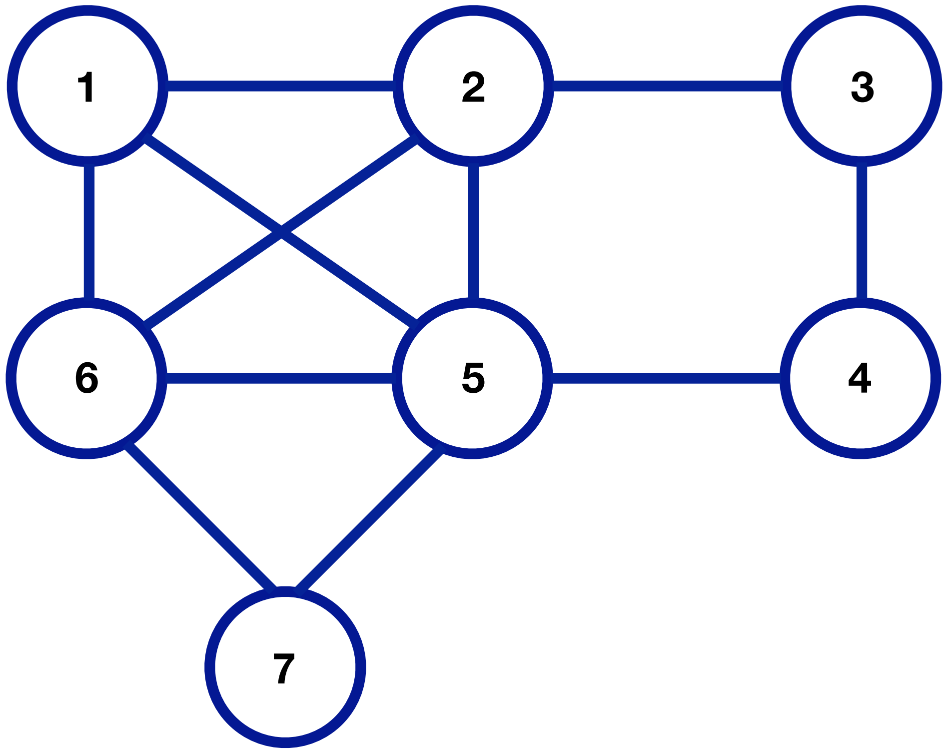
Can you find one? 1 3 7

Can you find two? 1 4 7

Can you find three? 2 4 7

Vertex Cover



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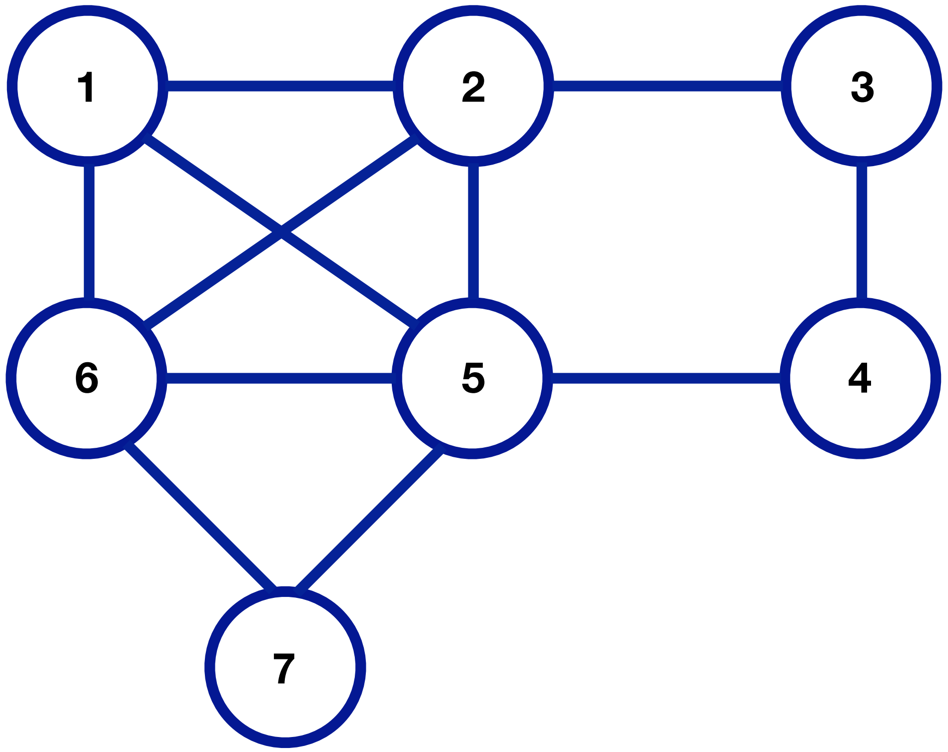

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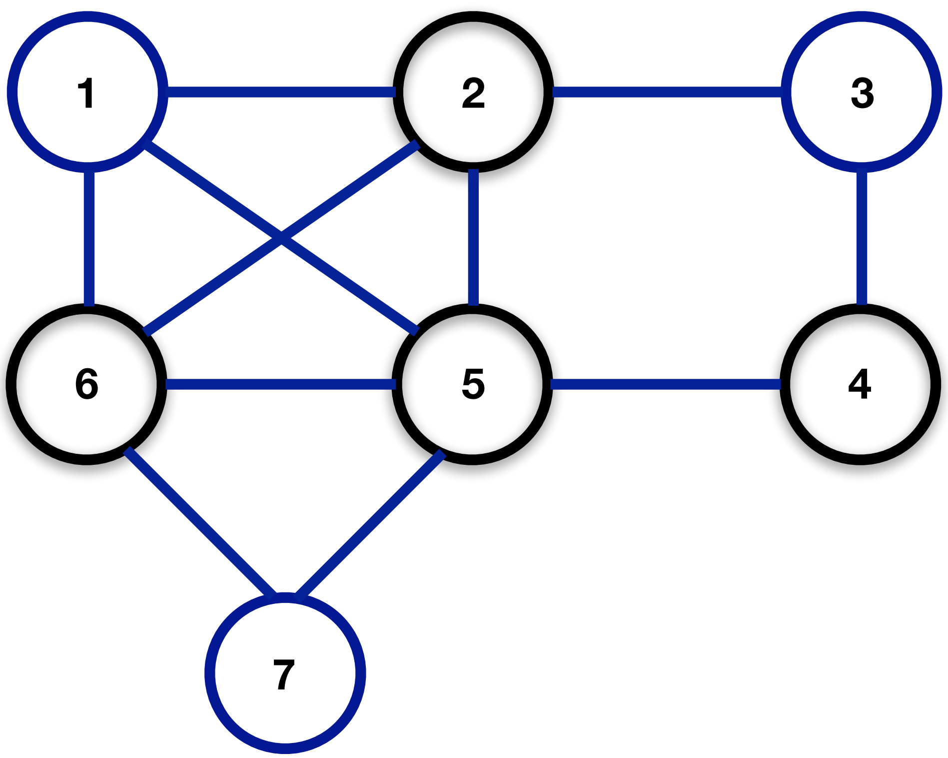
A **optimal vertex cover** is a vertex cover of minimum size for a given graph.

Can you find one?

Vertex Cover



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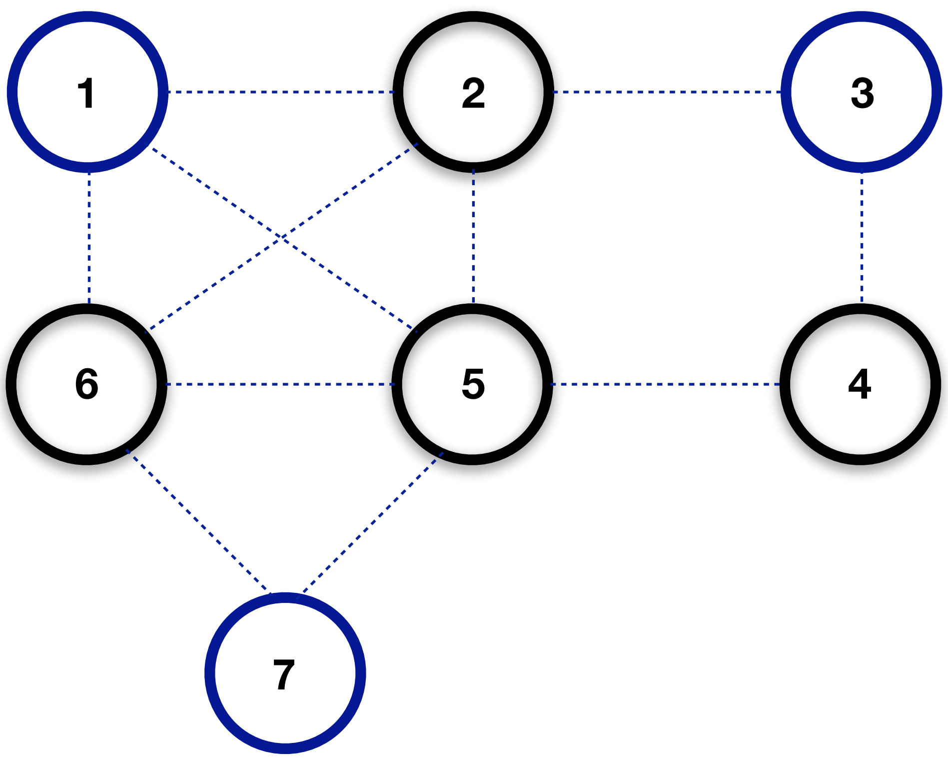
Can you find one? 2 4 5 6

Really?

Vertex Cover



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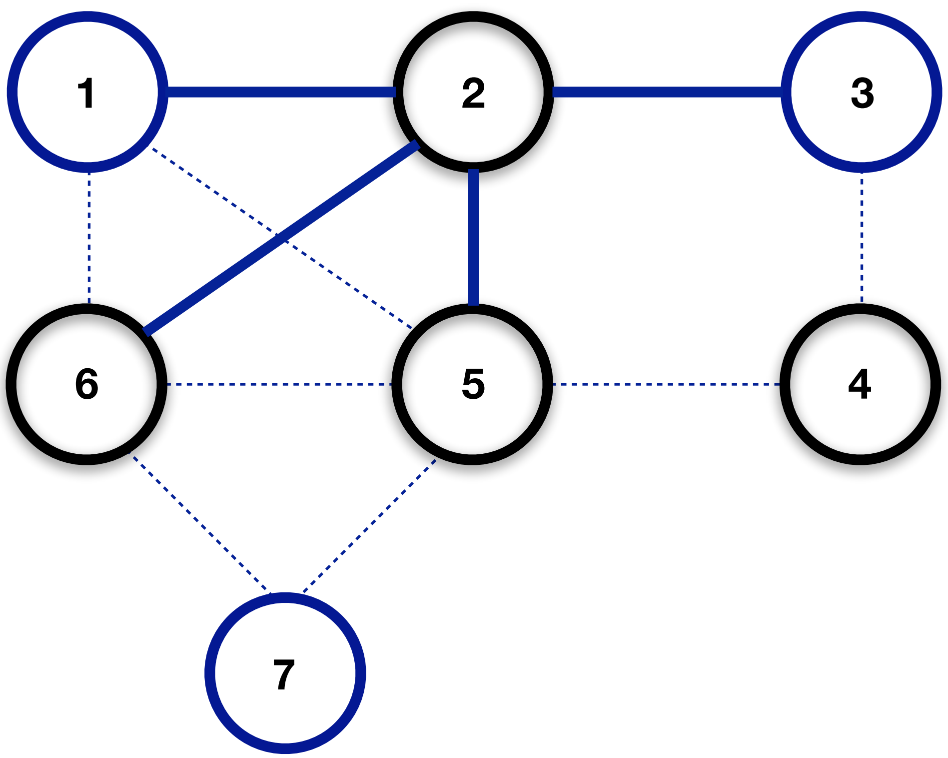
Can you find one? 2 4 5 6

Really? Let's test it.

Vertex Cover



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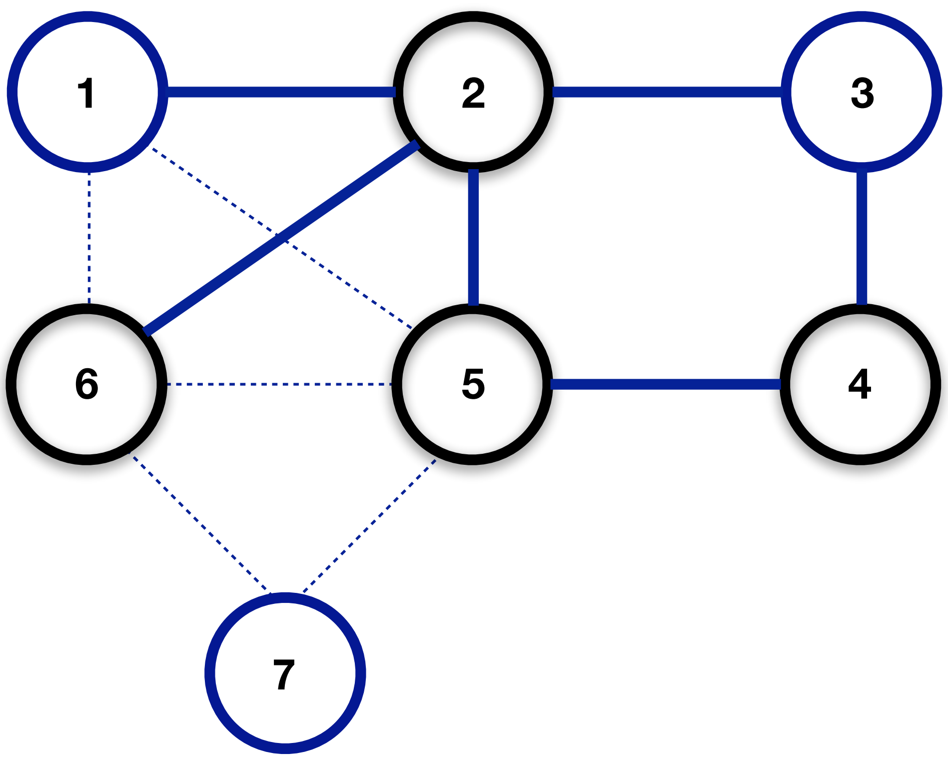
Can you find one? **2** 4 5 6

Really? Let's test it.
Vertex 2 covers 4 edges.

Vertex Cover



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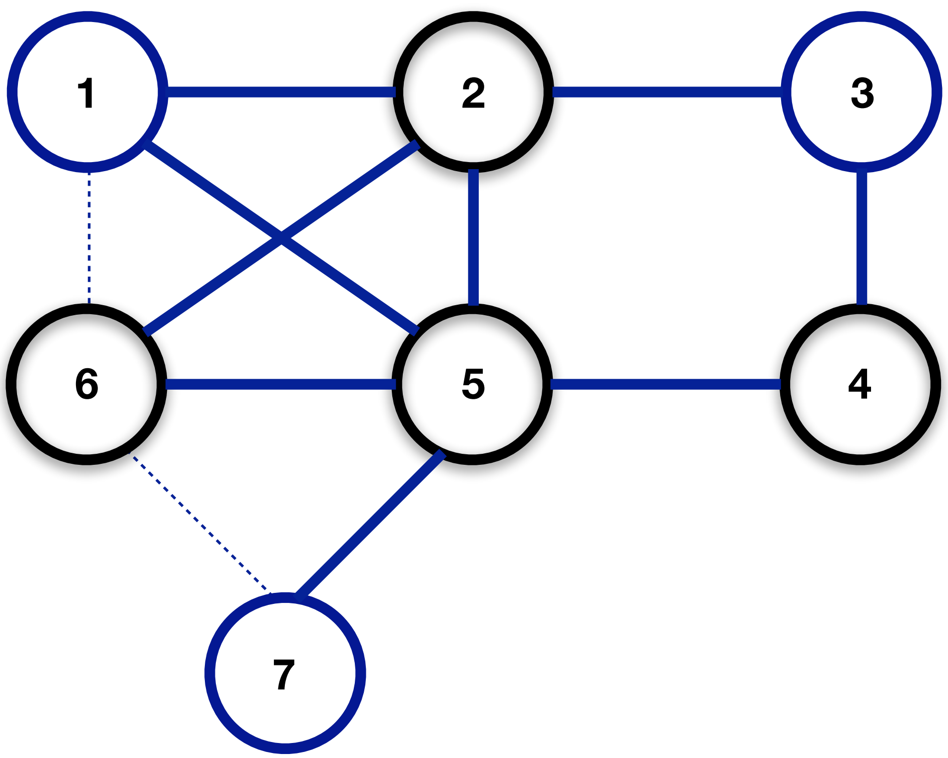
A **optimal vertex cover** is a vertex cover of minimum size for a given graph.

Can you find one? 2 **4** 5 6

Really? Let's test it.

Vertex 2 covers 4 edges.
Vertex 4 covers 2 more edges.

Vertex Cover



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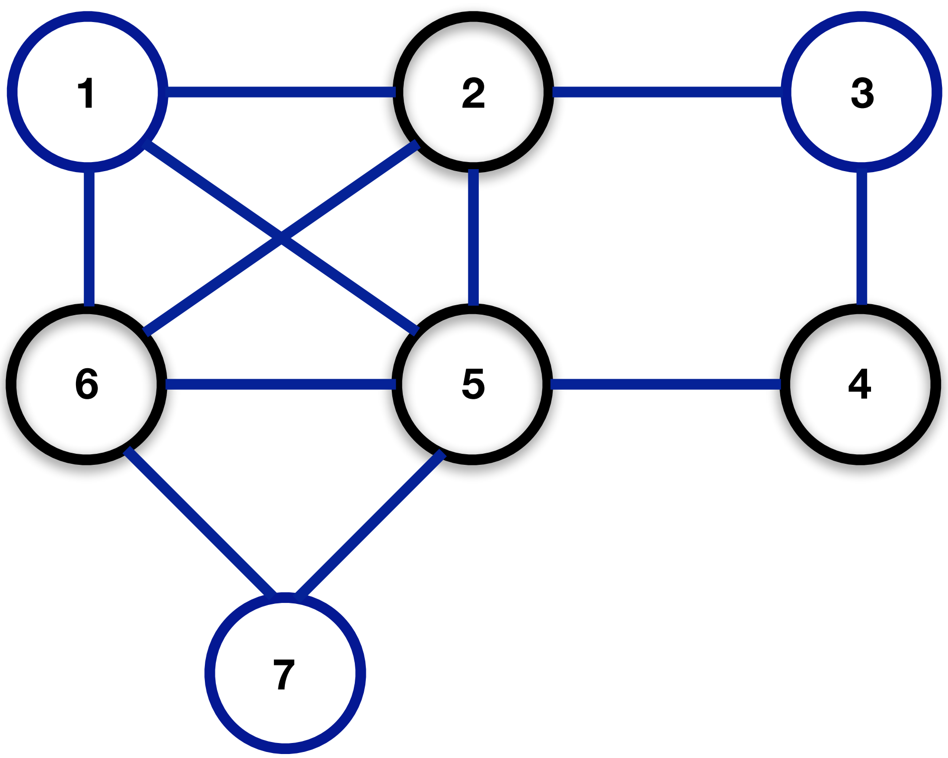
A **optimal vertex cover** is a vertex cover of minimum size for a given graph.

Can you find one? 2 4 **5** 6

Really? Let's test it.

Vertex 2 covers 4 edges.
Vertex 4 covers 2 more edges.
Vertex 5 covers 3 more edges.

Vertex Cover



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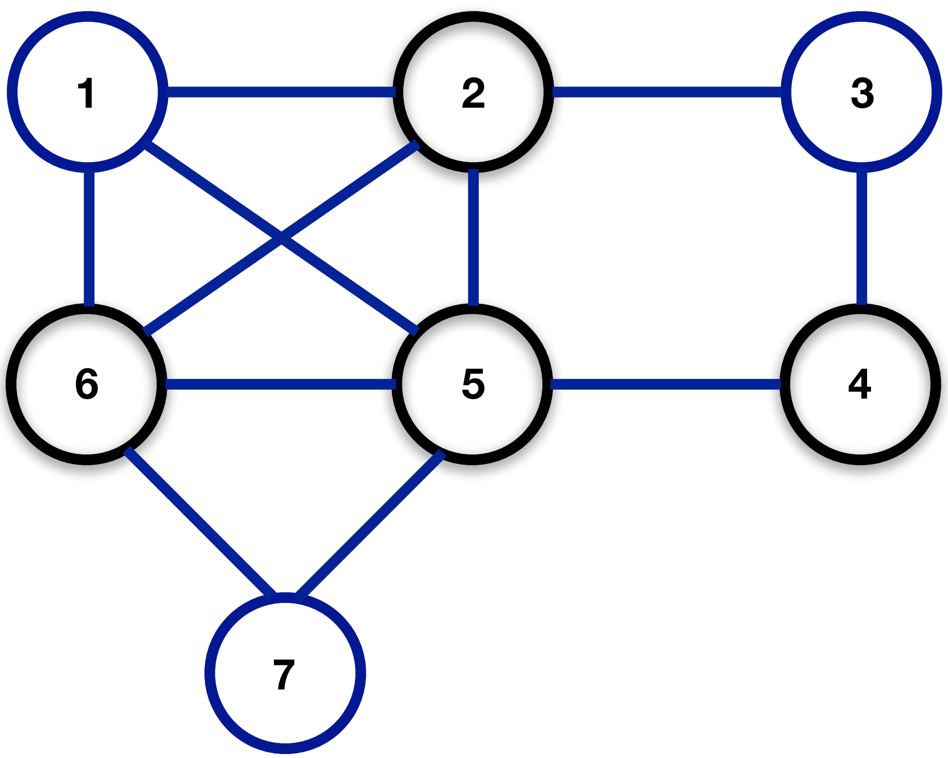
A **optimal vertex cover** is a vertex cover of minimum size for a given graph.

Can you find one? 2 4 5 **6**

Really? Let's test it.

Vertex 2 covers 4 edges.
Vertex 4 covers 2 more edges.
Vertex 5 covers 3 more edges.
Vertex 6 covers 2 more edges.

Vertex Cover



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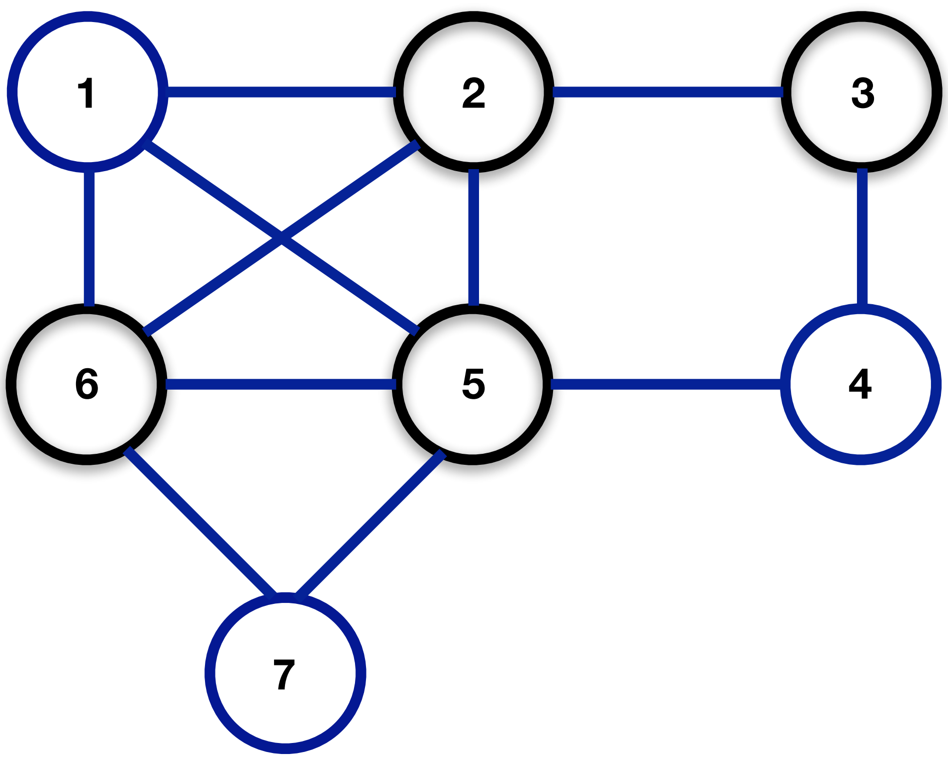
A **optimal vertex cover** is a vertex cover of minimum size for a given graph.

Can you find one? 2 4 5 6

Really? Let's test it.
Vertex 2 covers 4 edges.
Vertex 4 covers 2 more edges.
Vertex 5 covers 3 more edges.
Vertex 6 covers 2 more edges.
Yes, really.

Are there others?

Vertex Cover



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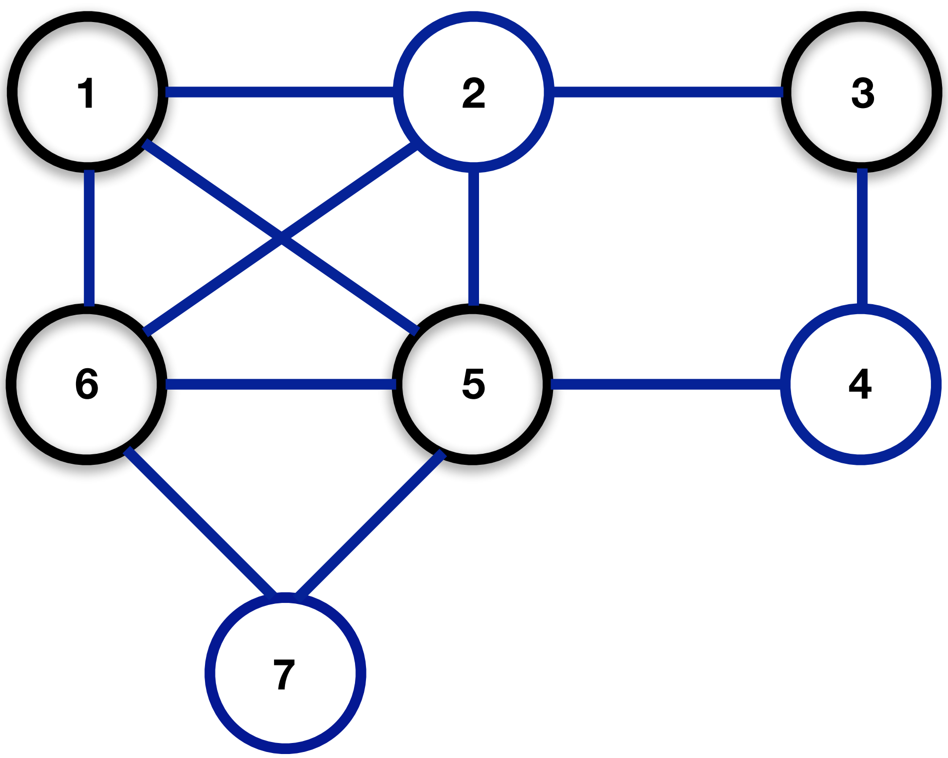
Can you find one? 2 4 5 6

Can you find two? 2 3 5 6

Can you find three?

Vertex Cover



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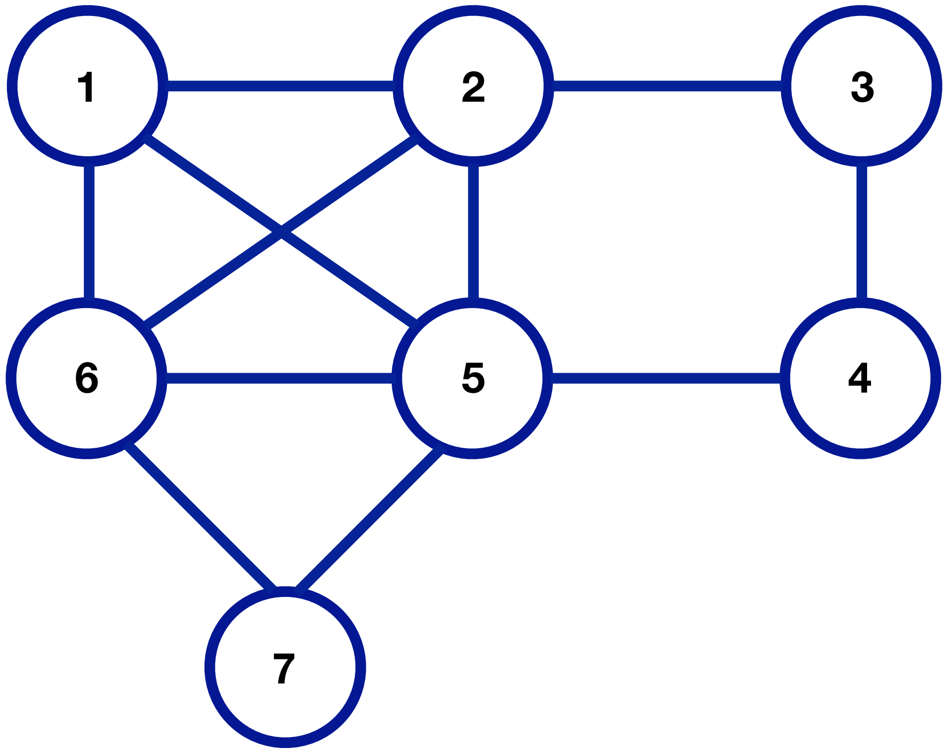
Can you find one? 2 4 5 6

Can you find two? 2 3 5 6

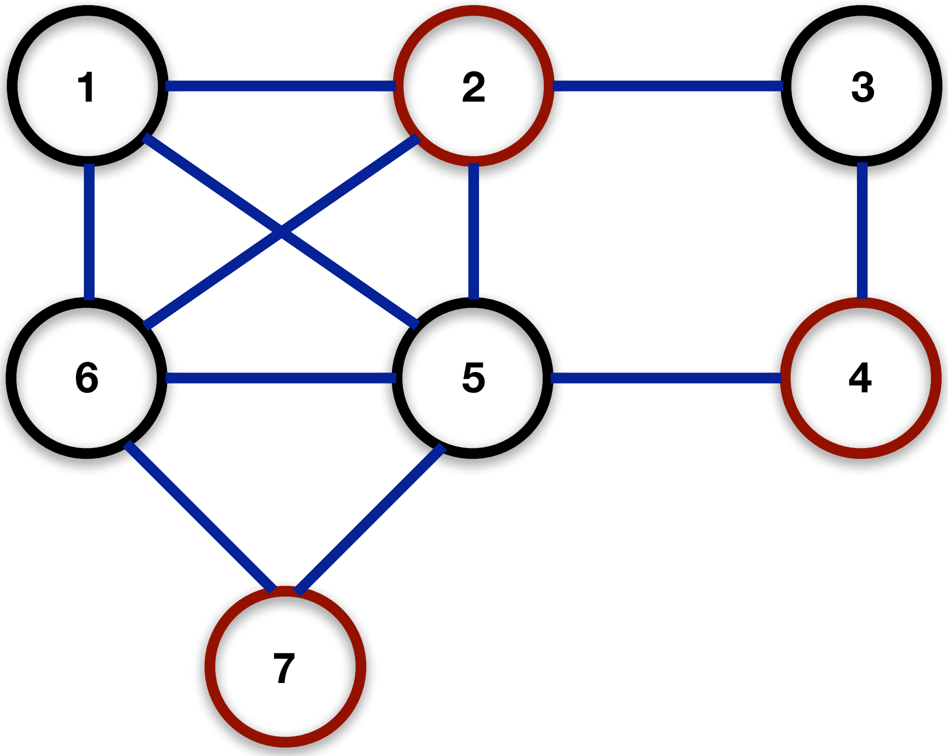
Can you find three? 1 3 5 6

Vertex Cover and Independent Sets



Did you notice a relationship between **vertex cover** and **independent set**?

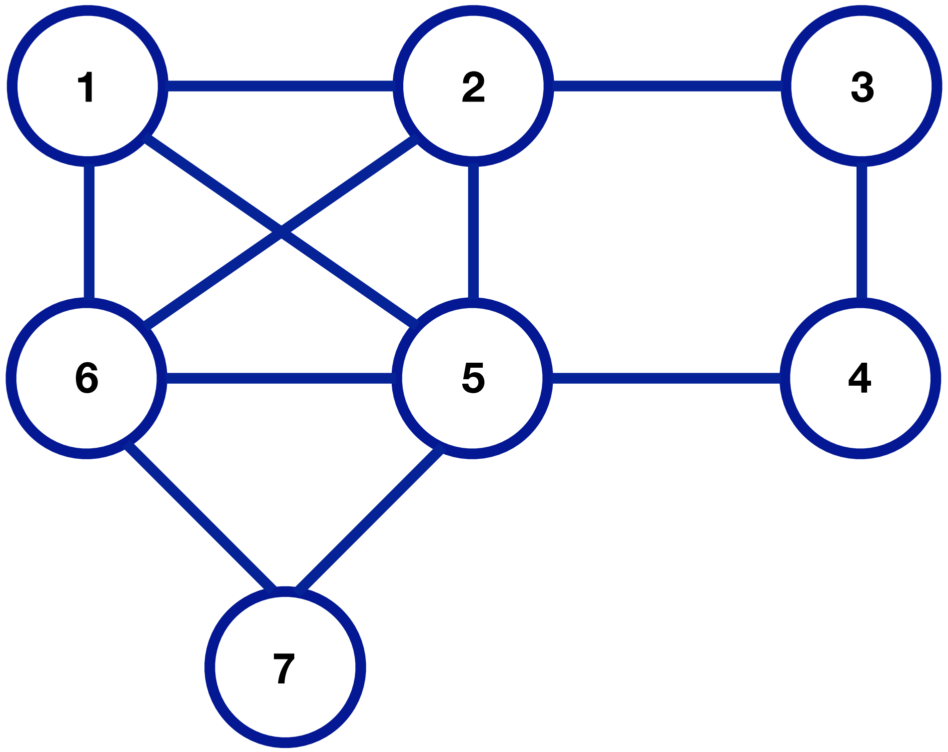
Vertex Cover and Independent Sets



Did you notice a relationship between **vertex cover** and **independent set**?

For any graph $G = (V, E)$, if $V' \subseteq V$ is a vertex cover for G then $V - V'$ is an independent set in G.

Cliques

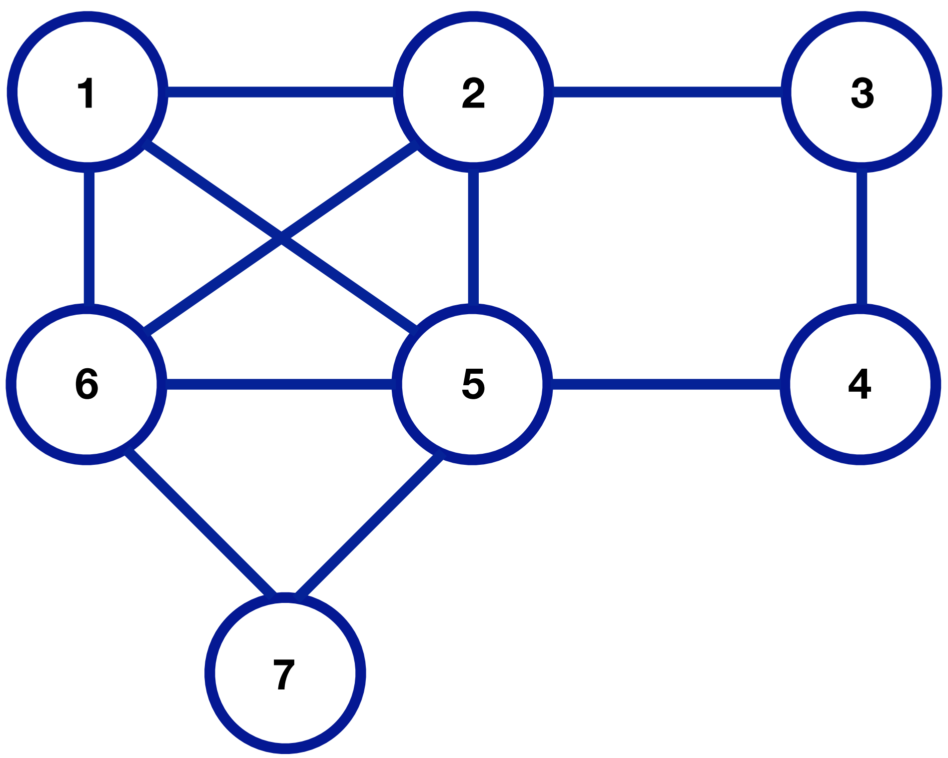


A **clique** in an undirected graph $G = (V, E)$

… is a subset of the vertex set $V' \subseteq V$, such that for every two vertices in V' there exists an edge connecting them.

… is the subgraph induced by V' as long as it is complete.

Cliques



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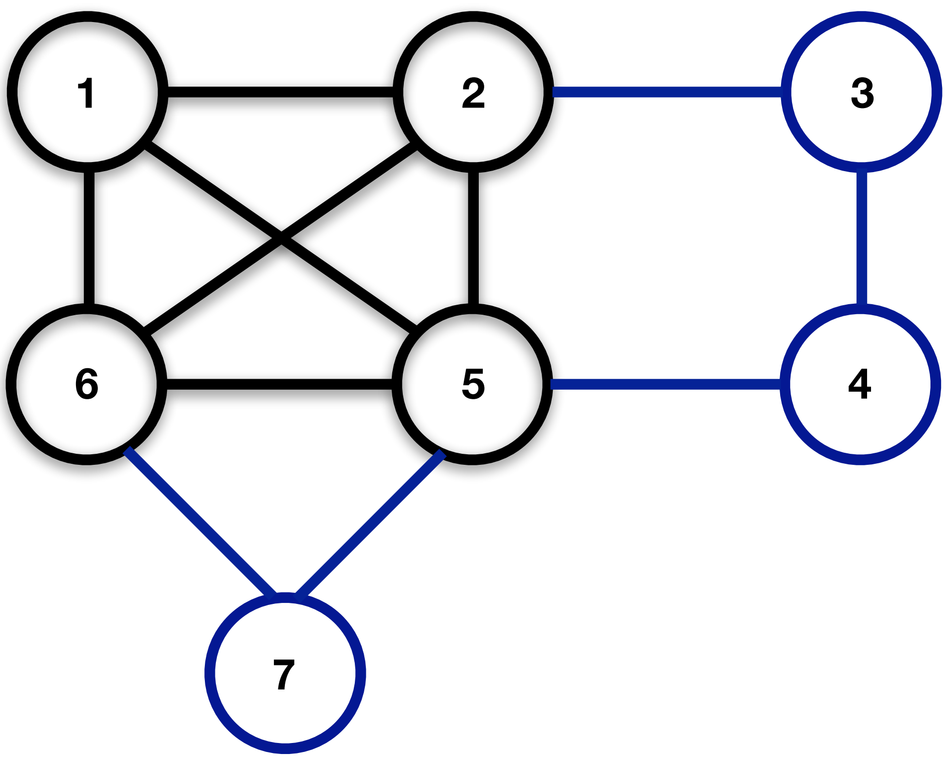
A **maximal clique** is a clique that cannot be extended by including even one more adjacent vertex.

Can you find a maximal clique?

Cliques



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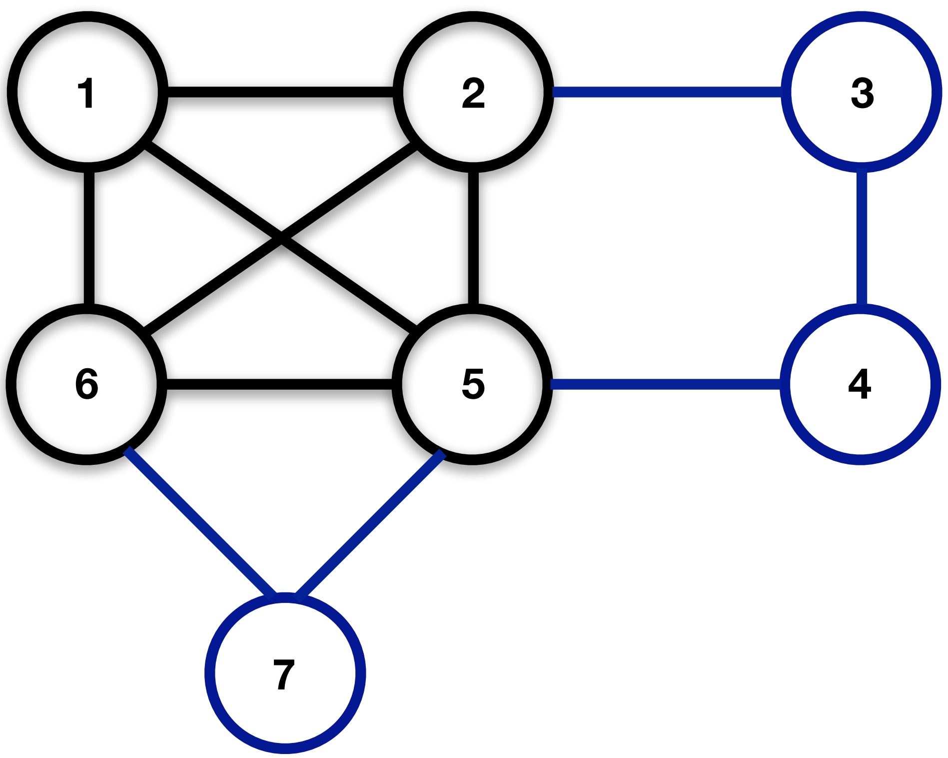
Can you find a maximal clique? 1 2 5 6

Cliques



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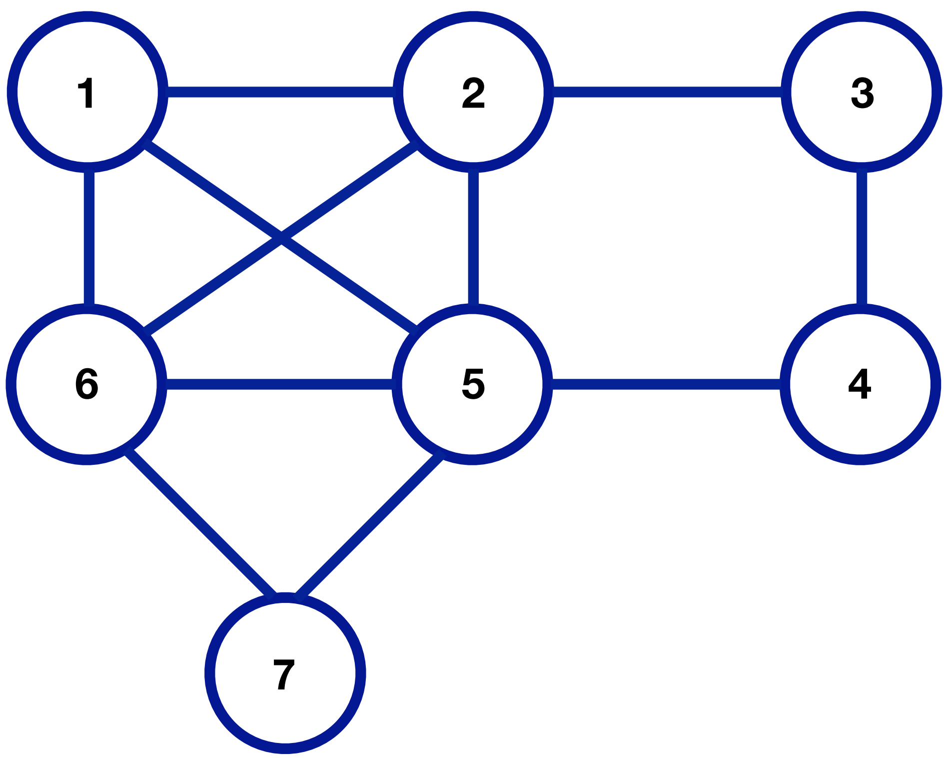
A **maximal clique** is a clique that cannot be extended by including even one more adjacent vertex.

Maximal clique: 1 2 5 6

These are highly connected vertices.
How connected?

Clustering Coefficient

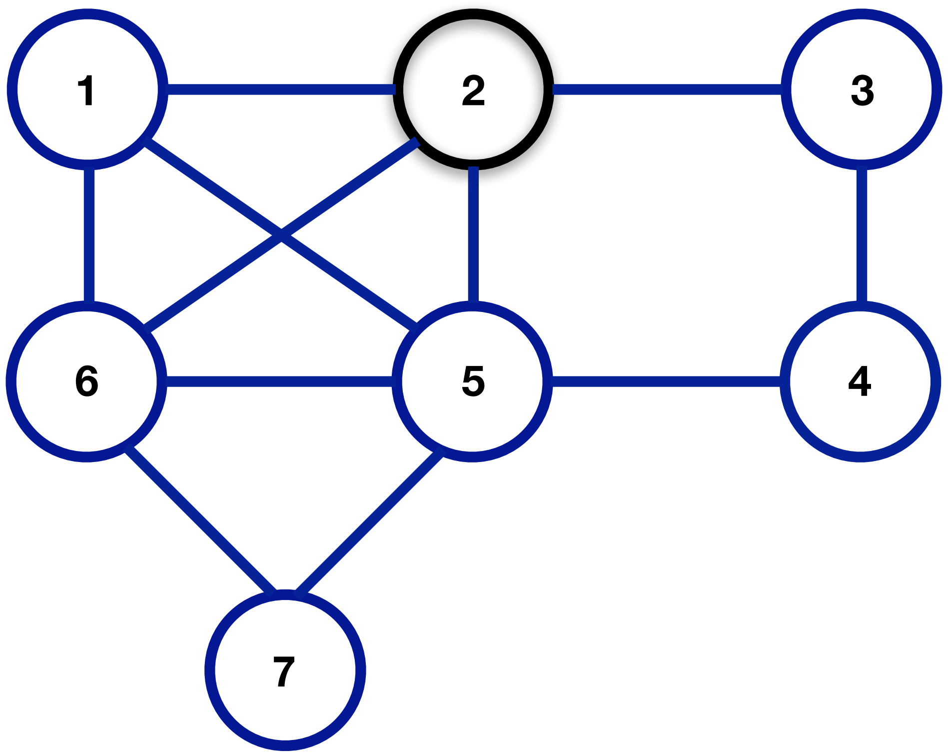


The **clustering coefficient** of a vertex in a graph is the probability that the neighbors of that vertex are also connected to each other.

It's a measure of the local connectivity or "clique-iness" of a (all or part of a) graph. (I.e., the probability that the adjacent vertices of a given vertex are connected.)

$$\frac{\text{number of actual edges among a vertex's neighbors}}{\text{number of possible edges among a vertex's neighbors}}$$

Clustering Coefficient



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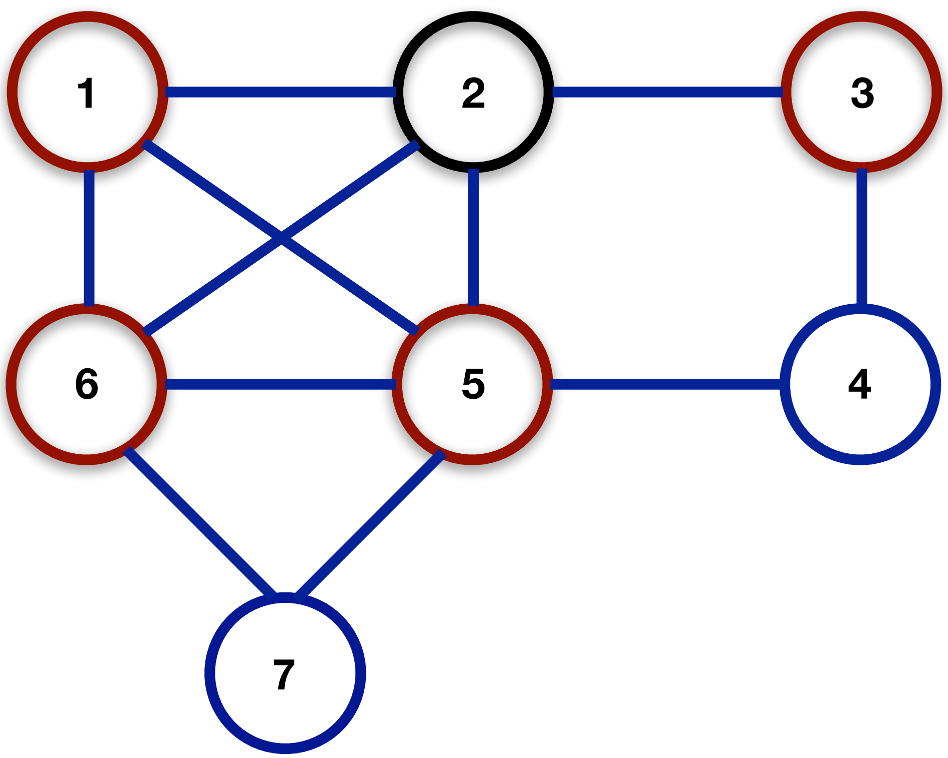
Let's calculate the clustering coefficient of vertex 2.

Clustering Coefficient



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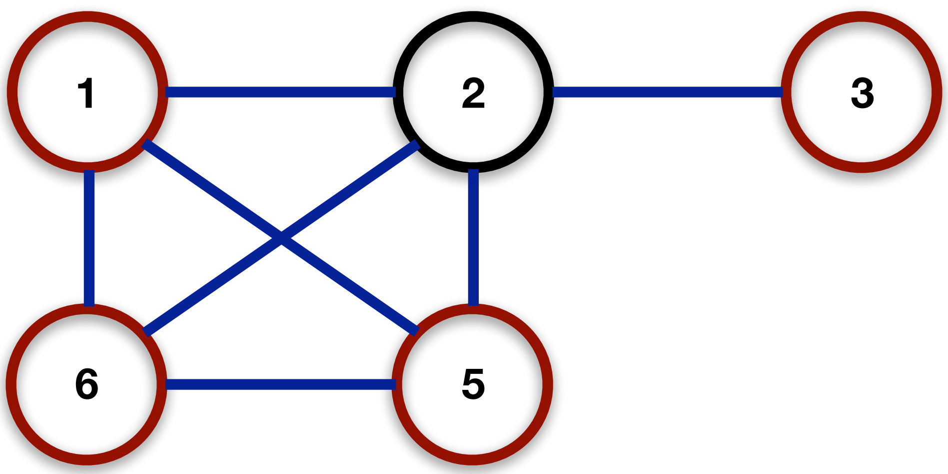
Let's calculate the clustering coefficient of vertex 2.

Vertex 2 is adjacent to vertices 1, 3, 5, and 6.

Clustering Coefficient



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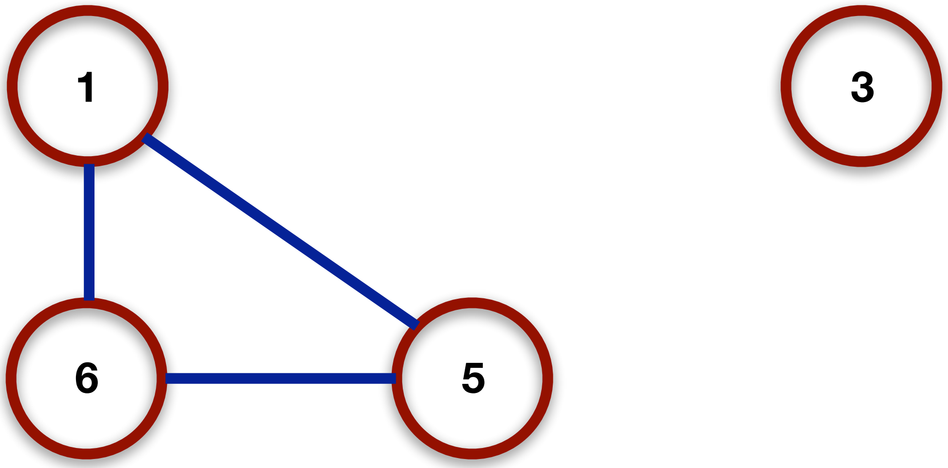
Let's calculate the clustering coefficient of vertex 2.

Vertex 2 is adjacent to vertices 1, 3, 5, and 6. Ignore the rest.

$$\frac{\text{number of actual edges among a vertex's neighbors}}{\text{number of possible edges among a vertex's neighbors}}$$

Clustering Coefficient



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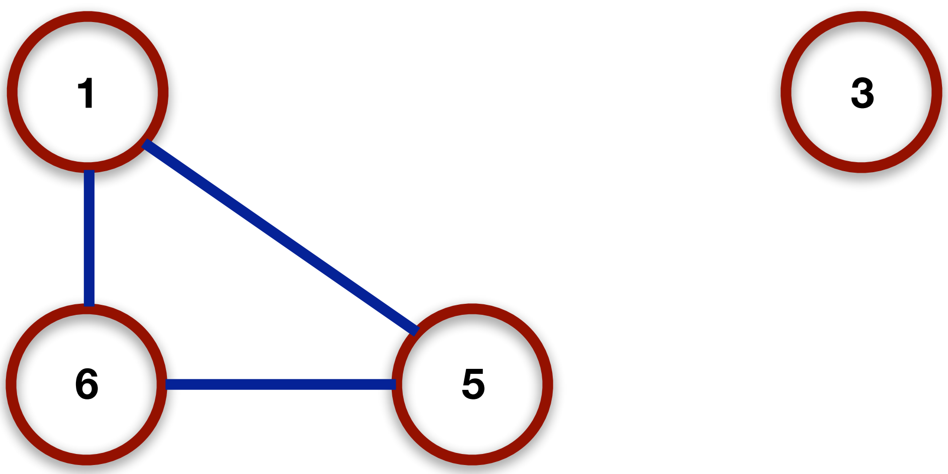
Let's calculate the clustering coefficient of vertex 2.

Vertex 2 is adjacent to vertices 1, 3, 5, and 6. Ignore the rest. Ignore vertex 2 itself, since this is about the neighbors.

$$\frac{\text{number of actual edges among a vertex's neighbors}}{\text{number of possible edges among a vertex's neighbors}}$$

Clustering Coefficient



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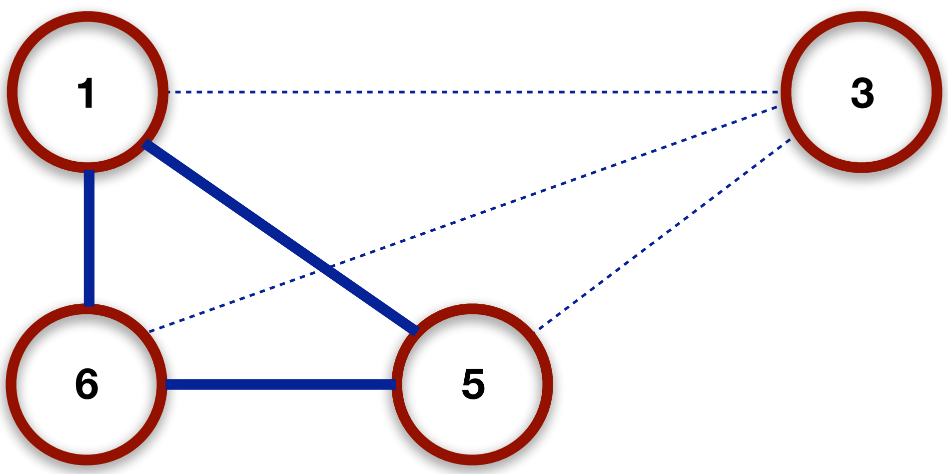
Let's calculate the clustering coefficient of vertex 2.

Vertex 2 is adjacent to vertices 1, 3, 5, and 6. Ignore the rest. Ignore vertex 2 itself, since this is about the neighbors.

There are 3 actual edges among the remaining vertices 1, 3, 5, and 6.

Clustering Coefficient



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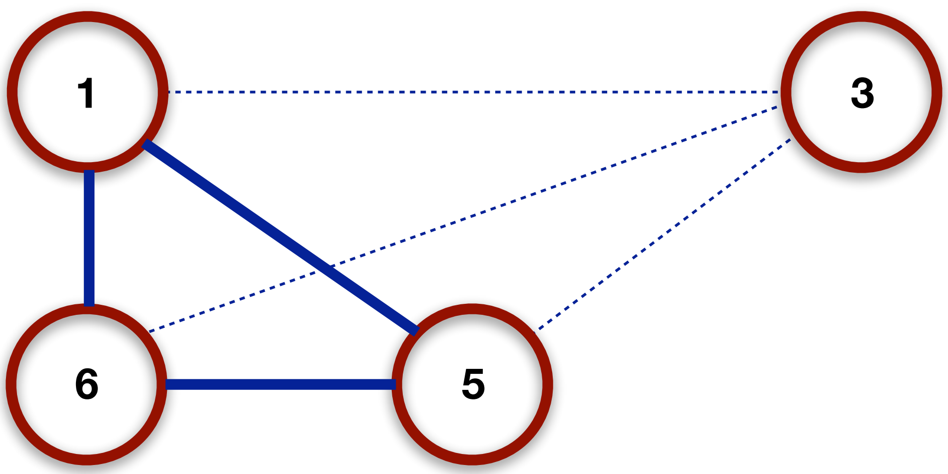
Let's calculate the clustering coefficient of vertex 2.

Vertex 2 is adjacent to vertices 1, 3, 5, and 6. Ignore the rest. Ignore vertex 2 itself, since this is about the neighbors.

There are 3 actual edges among the remaining vertices 1, 3, 5, and 6.

There are $n(n-1)/2$ possible edges among the remaining vertices 1, 3, 5, and 6 = 12/2 = 6.

Clustering Coefficient



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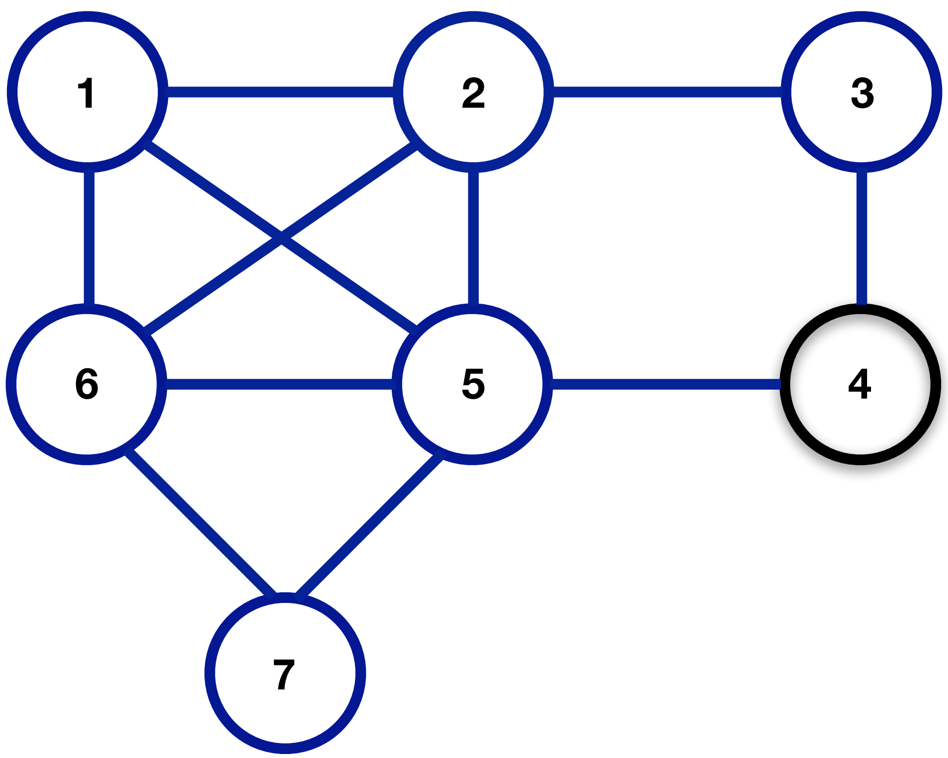
There are 3 actual edges among the remaining vertices 1, 3, 5, and 6.

There are $n(n-1)/2$ possible edges among the remaining vertices 1, 3, 5, and 6 = 12/2 = 6.

ClusteringCoefficient(vertex 2) = 3/6 = 0.5

Clustering Coefficient



$$\frac{\text{number of actual edges among a vertex's neighbors}}{\text{number of possible edges among a vertex's neighbors}}$$

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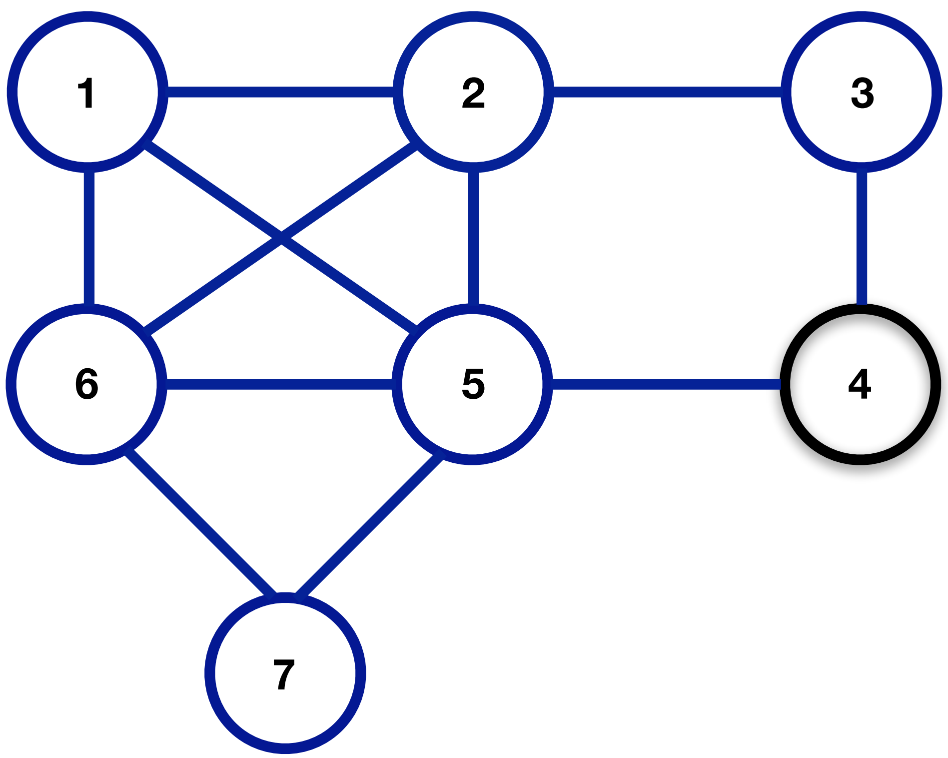
What's the clustering coefficient of vertex 4?

Clustering Coefficient



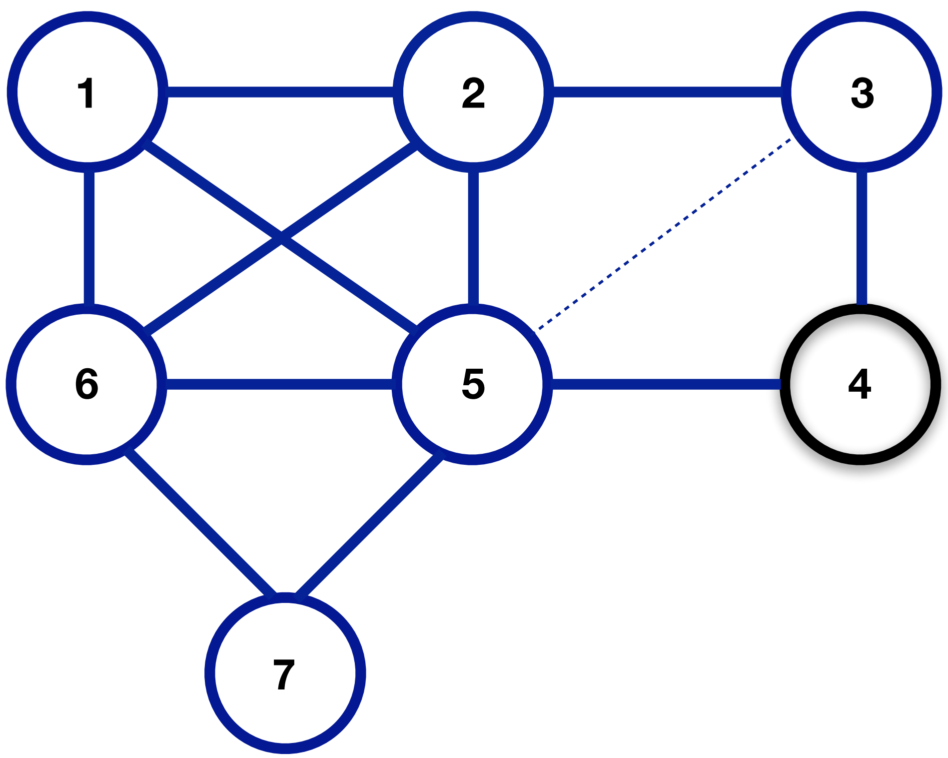
$$\frac{\text{number of actual edges among a vertex's neighbors}}{\text{number of possible edges among a vertex's neighbors}}$$

The **clustering coefficient** of a vertex in a graph is the probability that the neighbors of that vertex are also connected to each other.

It's a measure of the local connectivity or "clique-iness" of a (all or part of a) graph. (I.e., the probability that the adjacent vertices of a given vertex are connected.)

What's the clustering coefficient of vertex 4? It's 0.

Clustering Coefficient



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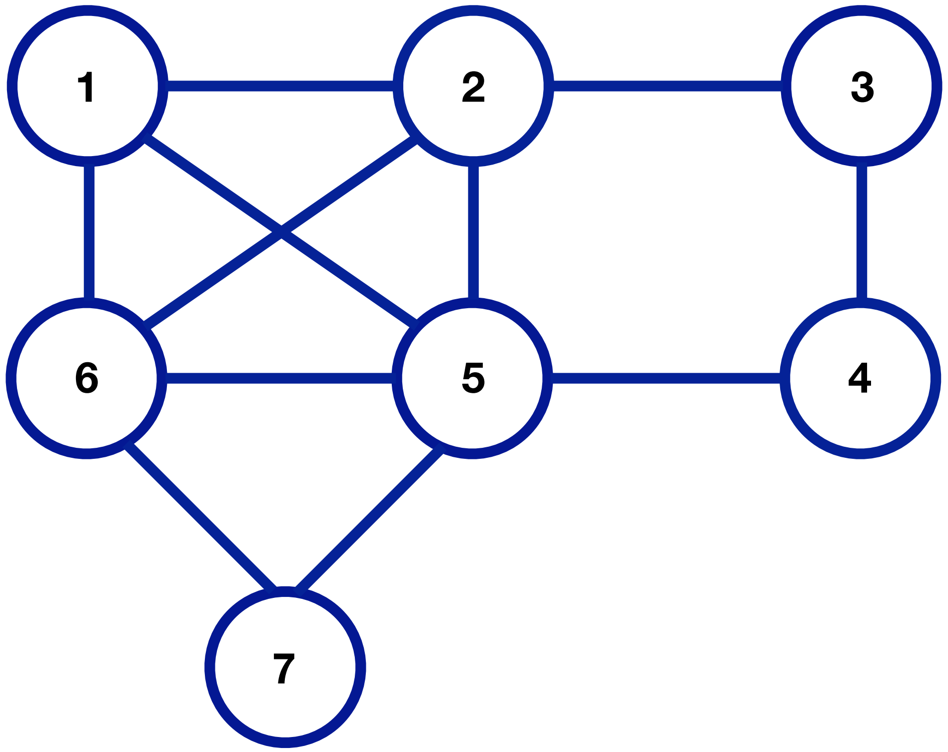
What's the clustering coefficient of vertex 4? It's 0.

We could increase it — and this make the graph more "clique-ie" — by closing the 3-4-5 triangle (math pun intended).

This **Triadic Closure** is a common graph operation, and what social networks do to suggest people you may know.

Clustering Coefficient



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How about the rest?

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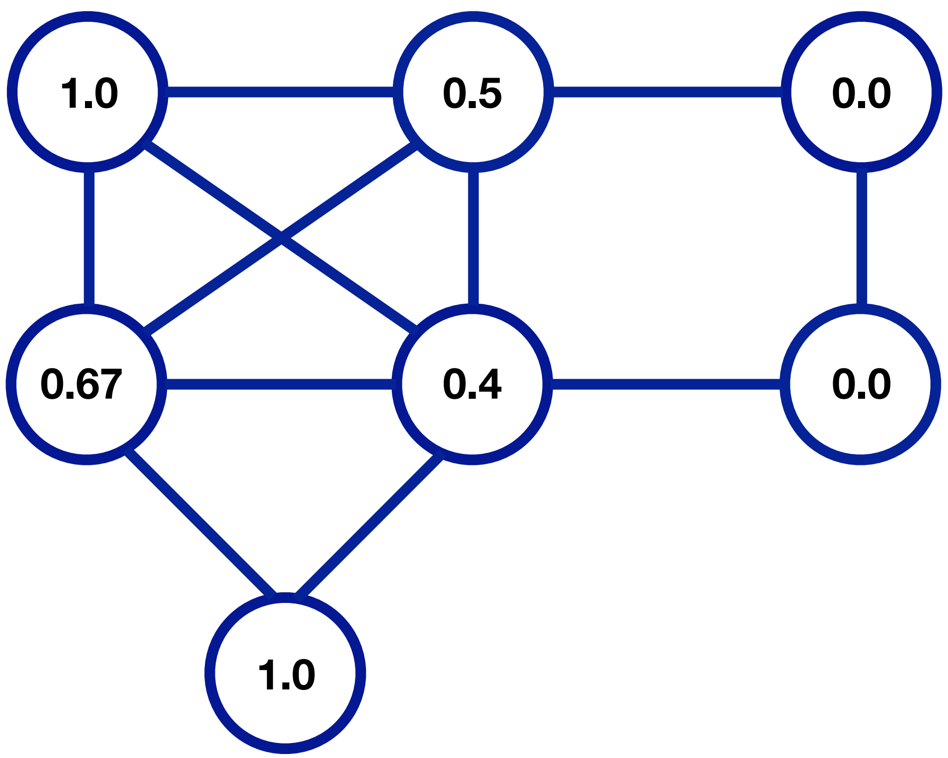

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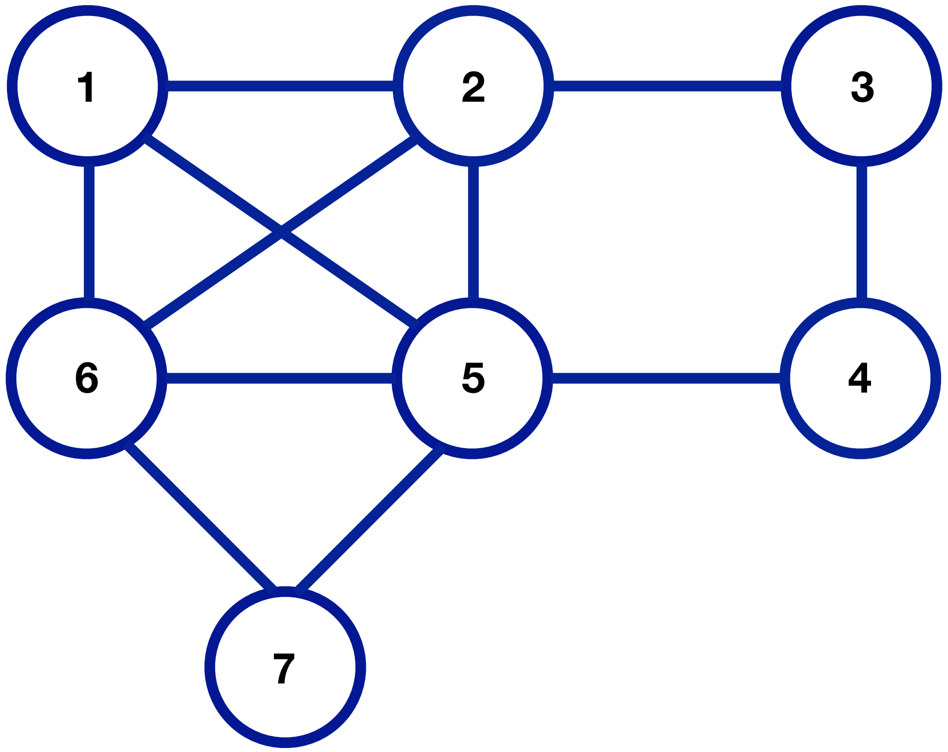
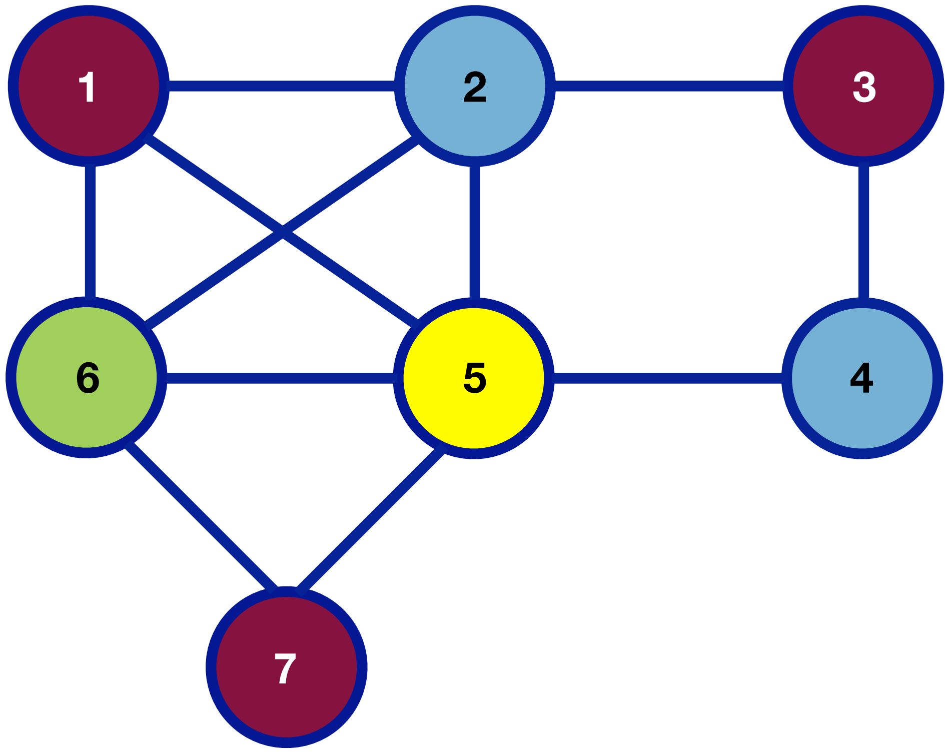
Graph Coloring



A **graph coloring** is a way of coloring the vertices of a graph such that no neighbors vertices share the same color.

The graph coloring problem is accomplishing this with a few colors as possible.

Graph Coloring



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Here is a 4-color solution.

Is there a 3-color solution?

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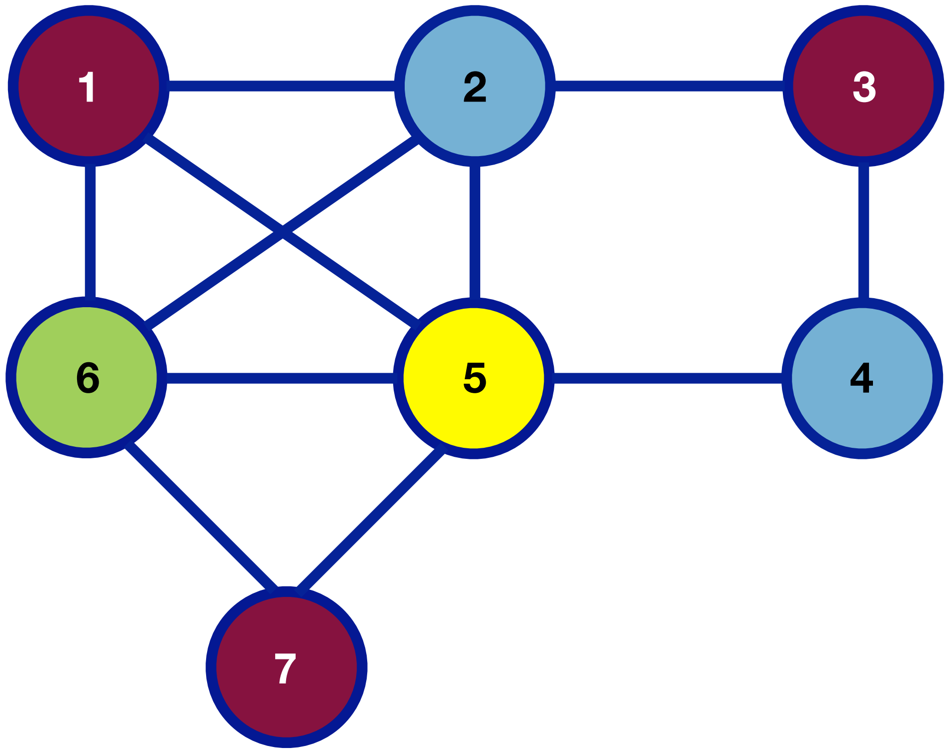

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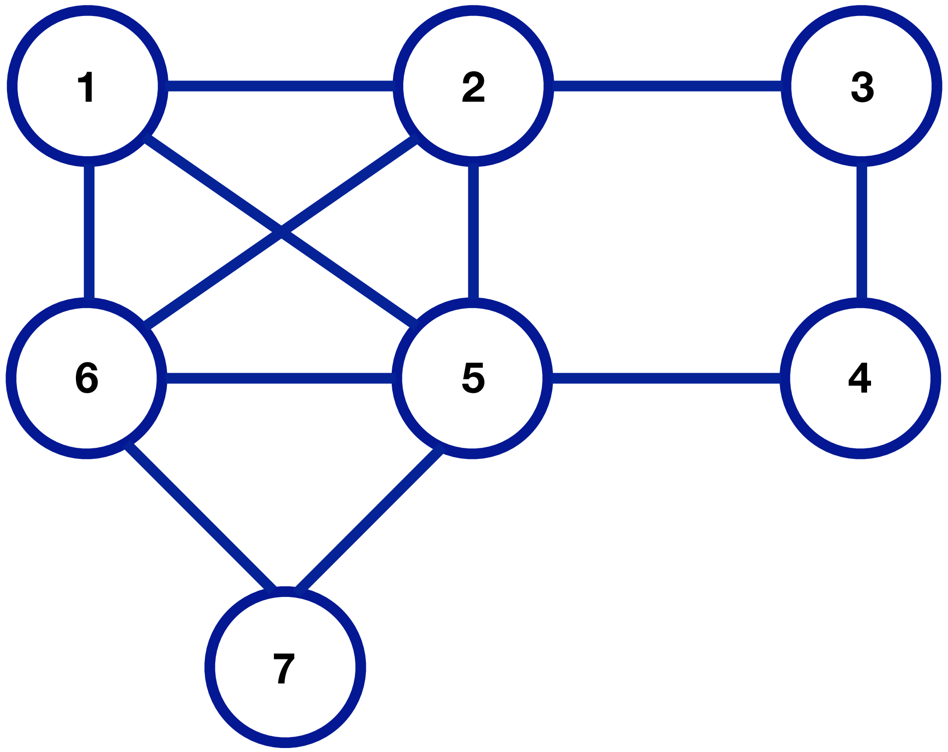
We can use graph coloring to model Sudoku.

There's more . . .



But it will have to wait.

- Distribution of vertex degrees
- Distribution of clustering coefficients and triadic closure
- Network density
- Size of connected components
- Shortest distance between pairs of vertices
- The centrality or eccentricity of vertices by various measures (PageRank and closeness centrality are of particular interest.)