Sorting - part one

Alan G. Labouseur, Ph.D.
Alan.Labouseur@Marist.edu
Total Order / Linear Order

From Appendix B in our CLRS text:

A relation $R$ on a set $A$ is a total relation if for all $a, b \in A$, we have $a R b$ or $b R a$ (or both), that is, if every pairing of elements of $A$ is related by $R$. A partial order that is also a total relation is a total order or linear order.

Let’s consider the relation ($R$) of $\leq$

A total order on $\leq$ is a binary relation that satisfies ...

- **totality** - either $a \leq b$ or $b \leq a$ or both.
- **transitivity** - if $a \leq b$ and $b \leq c$ then $a \leq c$.
- **anti-symmetry** - if $a \leq b$ and $b \leq a$ then $a = b$. 

Total Order / Linear Order

From Appendix B in our CLRS text:

A relation $R$ on a set $A$ is a **total relation** if for all $a, b \in A$, we have $a R b$ or $b R a$ (or both), that is, if every pairing of elements of $A$ is related by $R$. A partial order that is also a total relation is a **total order** or **linear order**. For example, the relation “$\leq$” is a total order on the natural numbers, but the “is a descendant of” relation is not a total order on the set of all people, since there are individuals neither of whom is descended from the other.

**Example: natural numbers:**
A total order on $\leq$ is a binary relation that satisfies ...

- **totality** $0 \leq 1 \leq 2 \leq 3 \leq 4 \ldots$
- **transitivity** $1 \leq 2$ and $2 \leq 3$ so $1 \leq 3$
- **anti-symmetry** the only way $a \leq b$ and $b \leq a$ is if $a = c$
  i.e., $1 \leq 1$ and $1 \leq 1$ or $2 \leq 2$ and $2 \leq 2$
Total Order / Linear Order

From Appendix B in our CLRS text:

A relation $R$ on a set $A$ is a total relation if for all $a, b \in A$, we have $a R b$ or $b R a$ (or both), that is, if every pairing of elements of $A$ is related by $R$. A partial order that is also a total relation is a total order or linear order. For example, the relation “$\leq$” is a total order on the natural numbers, but the “is a descendant of” relation is not a total order on the set of all people, since there are individuals neither of whom is descended from the other.

Counter-example: the “is a descendant of” relationship

- totality - Not everybody is related, so this violates totality.
Total Order / Linear Order

From Appendix B in our CLRS text:

A relation $R$ on a set $A$ is a total relation if for all $a, b \in A$, we have $a R b$ or $b R a$ (or both), that is, if every pairing of elements of $A$ is related by $R$. A partial order that is also a total relation is a total order or linear order. For example, the relation “$\leq$” is a total order on the natural numbers, but the “is a descendant of” relation is not a total order on the set of all people, since there are individuals neither of whom is descended from the other.

Counter-example: the “Rock, Paper, Scissors” relationships

- **transitivity** - scissors $<$ stone and stone $<$ paper, but scissors $\not<$ paper so this violates transitivity. In fact, scissors $> \text{ paper}$.
Total Order / Linear Order

From Appendix B in our CLRS text:

A relation $R$ on a set $A$ is a **total relation** if for all $a, b \in A$, we have $a R b$ or $b R a$ (or both), that is, if every pairing of elements of $A$ is related by $R$. A partial order that is also a total relation is a **total order** or **linear order**. For example, the relation “$\leq$” is a total order on the natural numbers, but the “is a descendant of” relation is not a total order on the set of all people, since there are individuals neither of whom is descended from the other.

**Counter-example: the “predator, prey” relationships**

- anti-symmetry - equal ferocity but different species.
- I.e., we have **anti-symmetry unless** $a \leq b$ and $b \leq a$ and $a \neq b$. 
## Permutations

Order matters

<table>
<thead>
<tr>
<th>set size</th>
<th>permutations</th>
<th>examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>{ (a) }</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>{ (a,b), (b,a) }</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>{ (a,b,c), (a,c,b), (b,a,c), (b,c,a), (c,a,b), (c,b,a) }</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>{ (a,b,c,d), (a,b,d,c), \ldots }</td>
</tr>
<tr>
<td>5</td>
<td>120</td>
<td>{ (a,b,c,d,e), \ldots }</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>

?
## Permutations

### Order matters

<table>
<thead>
<tr>
<th>set size</th>
<th>permutations</th>
<th>examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>{ (a) }</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>{ (a,b), (b,a) }</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>{ (a,b,c), (a,c,b), (b,a,c), (b,c,a), (c,a,b), (c,b,a) }</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>{ (a,b,c,d), (a,b,d,c), \ldots }</td>
</tr>
<tr>
<td>5</td>
<td>120</td>
<td>{ (a,b,c,d,e), \ldots }</td>
</tr>
<tr>
<td></td>
<td></td>
<td>\ldots</td>
</tr>
<tr>
<td>$n$</td>
<td></td>
<td>$n!$</td>
</tr>
</tbody>
</table>
Shuffle sort / Bogo sort / Monkey sort

procedure sort(in out list D)
begin
    boolean done := false;
    while (not done)
        randomly permute D
        if (D is sorted)
            done := true
        end if
    end while
    // D is returned out
end procedure
Shuffle sort / Bogo sort / Monkey sort

procedure sort(in out list D)
begin
    boolean done := false;
    while (not done)
        randomly permute D
        if (D is sorted)
            done := true
        end if
    end while
    // D is returned out
end procedure

In terms of $n$, the number of items in list D...

How many times through the loop until we expect it to be sorted?

How long do we expect each iteration to take?
Shuffle sort / Bogo sort / Monkey sort

procedure sort(in out list D)
begin
    boolean done := false;
    while (not done)
        randomly permute D
        if (D is sorted)
            done := true
        end if
    end while
    // D is returned out
end procedure

In terms of \( n \), the number of items in list D...

How many times through the loop until we expect it to be sorted? \( n! \)

How long do we expect each iteration to take?

- permute = \( O(n) \)
- check if sorted = \( O(n) \)

Total time = time per iteration \( \times \) number of iterations = \( O(n \times n!) \)
Shuffle sort / Bogo sort / Monkey sort

This is silly. And terrible.

But the worst part is that this is the **expected** case. The worst case scenario is that it never halts because there is no guarantee that we’ll ever produce a sorted list through random permutations. In that sense, it’s scary.

To put it more technically: \( O(scary) \)

Let’s not do this.

**Total time = time per iteration \( \times \) number of iterations = \( O(n \times n!) \)**
Selection Sort