

# Theory of Programming Languages

CMPT 331 • Fall 2019

—Fun with Mbnceb Dbmdvmvt - 100 points —

Goal To appreciate the joy of the Lambda Calculus as a fundamental model of computation.

Problems *Each part of each problem is worth 10 points*<sup>1</sup>.

1. Beta-reduce the following expressions to their normal form:

a.  $(\lambda a \lambda y . y a) (z z)$

b.  $(\lambda x \lambda y . (x y)) (\lambda z . y)$

c.  $(\lambda x . (x x)) (\lambda y . (y y))$

d.  $K x y$

e.  $S K$

f.  $(S K) y y z$

g.  $K' y y z$

2. What is the normal form of  $S (K S) (K I)$ ?

3. Prove the following equivalencies by reducing each side to its normal form.

a.  $I = S K K$

b.  $S K K = K I I$

4. Given the definition of Church numerals below, what does  $(m n)$  do when  $m$  and  $n$  are Church numerals? For example  $(\bar{2} \bar{3})$ . It may be easier to work out as  $\lambda m \lambda n . (m n)$ . Show your work (or at least an example).

## Church Numerals

Let  $\bar{0} = \lambda f x . x$

Let  $\bar{1} = \lambda f x . (f x)$

Let  $\bar{2} = \lambda f x . (f (f x))$

Let  $\bar{3} = \lambda f x . (f (f (f x)))$

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Let  $\bar{n} = \lambda f x . (f^n . x)$

Let successor =  $\overline{\text{succ}} = \lambda n f x . n f (f x)$

$(\overline{\text{succ}} \bar{0}) \equiv (\lambda n f x . n f (f x)) (\bar{0})$

$[\bar{0}/n] \text{ in } \lambda f x . n f (f x)$

$\rightarrow_{\beta} \lambda f x . \bar{0} f (f x)$

$[(f x)/f] \text{ in } \lambda x . \bar{0} f$

$\rightarrow_{\beta} \lambda x . \bar{0} (f x)$

$\equiv \lambda x . (\lambda f x . x) (f x)$

$[(f x)/x] \text{ in } \lambda f x . x$

$\rightarrow_{\beta} \lambda f x . (f x)$

$\equiv \bar{1}$

Submitting Write your answers using LaTeX. (See template and resources on our web site.) **Print** your document and hand it to me any time on or before the class in which it is due. Remember to include your name. Also, **you must show all work**. This is college. If my request that you show all your work is surprising to you, please drop this class immediately.

<sup>1</sup> Yes, I know it adds up to 110 points. You can make a mistake or two and still get a good score. Feel the love.